The Community Noah Land Surface Model with Multi-Parameterization Options (Noah-MP)

Technical Description

Zong-Liang Yang, Xitian Cai, Gang Zhang, Ahmad A. Tavakoly, Qinjian Jin, Lisa H. Meyer, Xiaodan Guan

Center for Integrated Earth System Science, Department of Geological Sciences,
The University of Texas at Austin, Austin, TX, USA

8/19/2011

Contents

1. Phenology

In current version, the phenology module aims to: 1) determine the fraction of vegetation canopy which is buried by snow; 2) define the growing season.

1.1 Vegetation buried by snow

The thickness of canopy buried by snow is

$$d_b = \min\left\{\max(h_{snow} - h_{v,b}, 0), \left(h_{v,t} - h_{v,b}\right)\right\}$$

where h_{snow} is snow height, $h_{v,t}$ and $h_{v,b}$ are the height of canopy top and bottom, respectively. The fraction of canopy buried by snow

$$f_b = \frac{d_b}{\max\left\{10^{-6}, \left(h_{v,t} - h_{v,b}\right)\right\}}$$

Note that when $0 < h_{v,t} \le 0.5$, a critical snow depth ($h_{snow,c}$), at which the short vegetation is fully covered by snow, is calculated as

$$h_{snow,c} = h_{v,t} \bullet e^{-h_{snow}/0.1}$$

Then f_b is calculated by

$$f_b = \begin{cases} \dfrac{h_{snow}}{h_{snow,c}} & h_{snow} < h_{snow,c} \\ 1 & h_{snow} \ge h_{snow,c} \end{cases}.$$

Once the vegetation is buried by snow, the effective LAI and SAI are given as

$$ELAI = LAI \bullet \left(1 - f_b\right)$$

$$ESAI = SAI \bullet \left(1 - f_b\right)$$

1.2 Flag of Growing season.

In phenology module, we determine whether it is growing season (IGS=1) or not (IGS=0) by

$$IGS = \begin{cases} 1; & T_v > T_{\min} \\ 0; & T_v \le T_{\min} \end{cases}$$

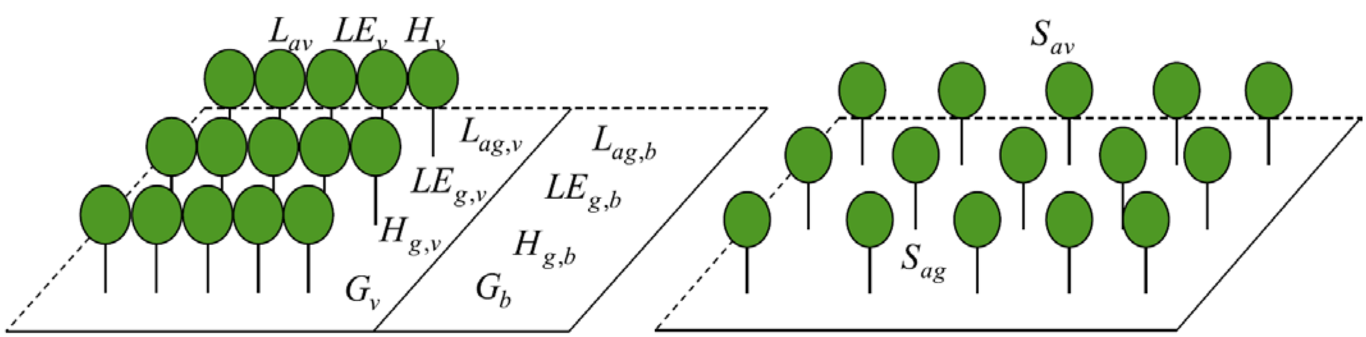
where $T_{\min}$ is the minimum temperature for photosynthesis for each vegetation type.

2. Energy

We use different approaches to deal with subgrid features of radiation transfer and turbulent transfer. We use 'tile' approach to compute turbulent fluxes, while we use modified two-stream to compute radiation transfer. Tile approach, assemblying vegetation canopies together, may expose too much ground surfaces (either covered by snow or grass) to solar radiation. The modified two-stream assumes vegetation covers fully the gridcell but with gaps between tree crowns.

Turbulence transfer: 'tile' approach to compute energy fluxes in vegetated fraction and bare fraction separately and then sum them up weighted by fraction.

Radiation transfer: modified two-stream [*Yang and Friedl*, 2003; *Niu ang Yang*, 2004] two-stream treats leaves as cloud over the entire grid-cell, while the modified two-stream aggregates cloudy leaves into tree crowns with gaps (as shown in the left figure). We assume these tree crowns are evenly distributed within the gridcell with 100% veg fraction, but with gaps. The 'tile' approach overlaps too much shadow.



1) Wind speed at reference height

$$U_r = \sqrt{u^2 + v^2} \geq 1 \tag{2-1}$$

Where U_r is wind speed at reference height (m s^{-1}). u is eastward component of wind speed (m s^{-1}). v is northward component of wind speed (m s^{-1}). (Why wind speed needs to be greater than 1 m s^{-1}?)

2) Vegetated or non-vegetated

$$PAI = LAI + SAI \tag{2-2}$$

Where PAI is plant area index (-). LAI is leaf area index (-). SAI is stem area index (-).

If PAI is greater than 0, then the surface is vegetated, otherwise, it is non-vegetated.

3) Ground snow cover fraction [*Niu ang Yang*, 2007]

$$\rho_{snow} = \frac{m_{snow}}{h_{snow}} \tag{2-3}$$

Where ρ_{snow} is bulk density of snow (kg m^{-3}). m_{snow} is snow mass (kg m^{-2}). h_{snow} is snow height (m).

$$f_{melt} = \left(\frac{\rho_{snow}}{\rho_{new}} \right)^m \tag{2-4}$$

Where f_{melt} is melting factor for snow cover fraction. ρ_{new} is fresh snow density (kg m^{-3}) (in *Niu ang Yang* [2007], not defined in the code), here use ρ_{new} = 100 kg m^{-3}. m is melting factor determining the curves in melting season, is adjustable depending on scale (generally, a larger value for a larger scale). It can be calibrated against observed snow cover fraction or surface albedo. Here use m = 1.0, which is different from the one in *Niu ang Yang* [2007] that estimated at 1.6 as calibrated against the AVHRR SCF data.

$$f_{snow} = \tanh\left(\frac{h_{snow}}{2.5 z_0 f_{melt}} \right) \tag{2-5}$$

Where f_{snow} is fractional area of the grid cell covered by snow, or snow cover fraction (-). z_{0g} the ground roughness length (m), here use z_{0g} = 0.01 m.

4) Ground roughness length

$$z_{0m,g} = \begin{cases} z_0 (1 - f_{snow}) + f_{snow} z_{0,snow} & \text{For soil} \\ 0.01 \times (1 - f_{snow}) + f_{snow} z_{0,snow} & \text{For lake, } T_g < T_{frz} \\ 0.01 & \text{For lake, } T_g \geq T_{frz} \end{cases} \tag{2-6}$$

Where $z_{0m,g}$ is ground roughness length for momentum (m). $z_{0,snow}$ is the snow surface roughness length (m), here use $z_{0,snow}$ = 0.002 m.

5) Roughness length and displacement height

$$z_{0m} = \begin{cases} z_{0m,vt} & \text{For vegetated surface} \\ z_{0m,g} & \text{For non-vegetated surface} \end{cases} \tag{2-7}$$

Where z_{0m} is roughness length for momentum (m). $z_{0m,vt}$ is roughness length for momentum determined by vegetation type (m).

$$d = \begin{cases} h_{snow} & \text{For non-vegetated, or vegetated with } h_{snow} > 0.65 h_{can} \\ 0.65 h_{can} & \text{For vegetated with } h_{snow} < h_{can} \end{cases} \tag{2-8}$$

Where d is the zero plane displacement (m). h_{can} is the top of canopy layer (m).

$$z_a = \max(d, h_{can}) + z'_a \tag{2-9}$$

Where z_a is the reference height (m). z'_a is the atmospheric received from the atmospheric model (m).

If $h_{snow} > z_a$, then

$$z_a = h_{snow} + z'_a \tag{2-10}$$

6) Canopy wind absorption coefficient

$$\alpha = \alpha_{vt} \tag{2-11}$$

Where α is the canopy wind speed extinction parameter (-) and α_{vt} is canopy wind speed extinction parameter from the vegetation lookup table.

7) Vegetation and ground emissivity

$$\varepsilon_v = 1 - e^{-(LAI+SAI)/\bar{\mu}} \tag{2-12}$$

Where ε_v is the vegetation emissivity (-). $\bar{\mu}$ is the average inverse optical depth for longwave radiation, here $\bar{\mu}$ =1.0.

$$\varepsilon_g = \begin{cases} \varepsilon_{soil}(1-f_{snow}) + \varepsilon_{snow} f_{snow} & \text{For soil} \\ \varepsilon_{lake}(1-f_{snow}) + \varepsilon_{snow} f_{snow} & \text{For lake} \end{cases} \tag{2-13}$$

Where ε_{soil}, ε_{lake}, and ε_{snow} are the emissivity for soil, lake, and snow respectively (-).

8) Soil moisture factor controlling stomatal resistance

We implemented three options for this factor: (1) Noah type using soil moisture, (2) CLM type using matric potential, and (3) SSiB type also using matric potential but expressed by a different function [*Xue et al.*, 1991]. The Noah-type factor is parameterized as a function of soil moisture. The CLM-type factor [*Oleson et al.*, 2004] is a refined version of that of BATS [*Yang and Dickinson*, 1996].

$$\beta = \sum_{i=1}^{N_{root}} \frac{\Delta z_i}{z_{root}} \min\left(1.0, \frac{\theta_{liq,i} - \theta_{wilt}}{\theta_{ref} - \theta_{wilt}}\right) \quad \text{Noah type} \tag{2-14}$$

$$\beta = \sum_{i=1}^{N_{root}} \frac{\Delta z_i}{z_{root}} \min\left(1.0, \frac{\Psi_{wilt} - \Psi_i}{\Psi_{wilt} - \Psi_{sat}}\right) \quad \text{CLM type} \tag{2-15}$$

$$\beta = \sum_{i=1}^{N_{root}} \frac{\Delta z_i}{z_{root}} \min\left(1.0, 1.0 - e^{-c_2 \ln(\Psi_{wilt}/\Psi_i)}\right) \quad \text{CLM type} \tag{2-16}$$

Where θ_{wilt} and θ_{ref} are soil moisture at witling point (m^3 m^{-3}) and a reference soil moisture (m^3 m^{-3}) (close to field capacity), respectively. Both depend on soil type. N_{root} and z_{root} are total number of soil layers containing roots and total depth of root zone, respectively. $\Psi_i = (\theta_{liq,i}/\theta_{sat})^{-b}$ is the matric potential of the *i*th layer soil, and Ψ_{wilt} is the wilting matric potential, which is –150 m (equivalent to a 150 m-deep water table under steady state of soil hydrology) independent of vegetation and soil types. c_2 is a slope factor ranging from 4.36 for crops to 6.37 for broadleaf shrubs [see *Xue et al*., 1991, table 2]. When c_2=1, equation (14) [see Noah-MP paper *Niu et al*, 2011] becomes very close to equation (13). The CLM-type β factor shows a sharper and narrower range of variation with soil moisture than the Noah-type does (Figure 2). The SSiB β factor (c_2 = 5.8 in the figure) is even steeper than the CLM-type. These three options represent a great uncertainty in formulating the β factor in LSMs.

9) Soil surface resistance for ground evapotranspiration [*Sellers et al*., 1992]

$$r_{surf} = f_{snow} \times 1.0 + \left(1 - f_{snow}\right) e^{\left(8.206 - \alpha S_1\right)} \tag{2-17}$$

Where r_{surf} is the soil surface resistance (s m^{-1}). f_{snow} is the snow fraction covering a ground surface. S_1 is soil wetness in the top soil layer, varying from 0 to 1. α is a surface dryness factor controlling the effect of soil moisture on r_{surf}.

If $\theta_{liq,1} < 0.01$ and $h_{snow} = 0$, then

$$r_{surf} = 1.0 \times 10^{6} \text{ s m}^{-1} \tag{2-18}$$

$$\Psi = -\Psi_{sat} \left[\max\left(0.01, \frac{\theta_{liq,1}}{\theta_{sat}} \right) \right]^{-S_1} \tag{2-19}$$

Where Ψ is soil matric potential (mm). Ψ_{sat} is the saturated soil matric potential (mm).

$$RH = f_{snow} + \left(1 - f_{snow}\right) e^{\left(\frac{\Psi g}{R_w T_g}\right)} \tag{2-20}$$

Where RH is the relative humidity in surface soil/snow air space (-). Ψ is the soil matric potential (mm). Ψ_{sat} is the saturated soil matric potential (mm). R_w is the gas constant for water vapor (J kg^{-1} K^{-1}).

10) Set psychrometric constant

$$\lambda = \begin{cases} \lambda_{vap} & T > T_{frz} \\ \lambda_{sub} & T \le T_{frz} \end{cases} \tag{2-21}$$

Where λ is the latent heat of vaporization/sublimation (J kg^{-1}). λ_{vap} is the latent heat of vaporization (J kg^{-1}). λ_{sub} is the latent heat of sublimation (J kg^{-1}).

$$\gamma = \frac{C_{air} P_{atm}}{0.622 \lambda} \tag{2-22}$$

Where γ is the psychrometric constant (Pa K^{-1}). P_{atm} is the atmospheric pressure (Pa).

$$T_{rad} = \left(\frac{L\uparrow}{\sigma} \right)^{1/4} \tag{2-23}$$

Where T_{rad} is the radiative temperature (K). $L\uparrow$ is the upward longwave radiation (W m^{-2}). σ is the Stefan-Boltzmann constant (W m^{-2} K^{-4}).

$$PAR = PAR^{sun} L^{sun} + PAR^{sha} L^{sha} \tag{2-24}$$

Where PAR is the total photosynthetically active radiation (W m^{-2}). PAR^{sun} is the PAR absorbed per sunlit LAI (W m^{-2}). PAR^{sha} is the PAR absorbed per shaded LAI (W m^{-2}). L^{sun} and L^{sha} are the sunlit and shaded leaf area indices (-).

$$A = A^{sun} L^{sun} + A^{sha} L^{sha} \qquad (2\text{-}25)$$

Where A is the total photosynthesis (μ mol CO_2 m^{-2} s^{-1}). A^{sun} is the sunlit photosynthesis (μ mol CO_2 m^{-2} s^{-1}). A^{sha} is the shaded photosynthesis (μ mol CO_2 m^{-2} s^{-1}).

References

Niu, G.-Y. and Z.-L. Yang (2004), The effects of canopy processes on snow surface energy and mass balances, *Journal of Geophysical Research*, **109**, D23111, doi:10.1029/2004JD004884.

Niu, G.-Y., and Z.-L. Yang (2007), An observation-based formulation of snow cover fraction and its evaluation over large North American river basins, *J. Geophys. Res*., 112, D21101, doi:10.1029/2007JD008674.

Oleson, K. W., et al. (2004), Technical description of the Community Land Model (CLM), *NCAR Tech. Note NCAR/TN-461+STR*, 174 pp., Natl. Cent. for Atmos. Res., Boulder, Colo. (Available at www.cgd.ucar.edu/tss/clm/distribution/clm3.0/index.html.)

Sellers, P. J., M. D. Heiser, and F. G. Hall (1992), Relations between surface conductance and spectral vegetation indices at intermediate (100 m2 to 15 km2) length scales, *J. Geophys. Res.*, 97, 19,033–19,059, doi:10.1029/92JD01096.

Xue, Y., P. J. Sellers, J. L. Kinter, and J. Shukla (1991), A simplified biosphere model for global climate studies, *J. Clim*., 4, 345–364, doi:10.1175/1520-0442(1991)004<0345:ASBMFG>2.0.CO;2.

Yang, Z.-L., and R. E. Dickinson (1996), Description of the Biosphere-Atmosphere Transfer Scheme (BATS) for the soil moisture workshop and evaluation of its performance, *Global Planet. Change*, *13*, 117–134, doi:10.1016/0921-8181(95)00041-0.

Yang, R., and M.A. Friedl (2003), Modeling the effects of 3-D vegetation structure on surface radiation and energy balance in boreal forests, *Journal of Geophysical Research, Atmospheres*, 108 (D16), 8615, doi: 10.1029/2002JD003109.

3. Radiation Transfer in Canopy

3.1 Chapter Summary

This chapter addresses the parameterization of radiation transfer through vegetation canopy. Versatile Integrator of Surface Atmosphere (VISA) model, used in the National Center for Atmospheric Research Land Surface Model version 1.0 (NCAR LSM 1.0), incorporates two-stream radiation-transfer scheme which overestimates the downward sensible heat flux, therefore resulting in earlier snow melting and a shallower snowpack. In order to overcome this disadvantage, a modified two-stream radiation scheme (*Niu and Yang*, 2004) was implemented in Noah_MP land surface model.

Snow cover is very sensitive to the radiation transferred within canopy.

The radiation subroutine called two other subroutines which are used to calculate the surface (including canopy) albedo, fluxes (per unit incoming direct and diffuse radiation) reflected, transmitted, and absorbed by vegetation, and sunlit fraction of the canopy.

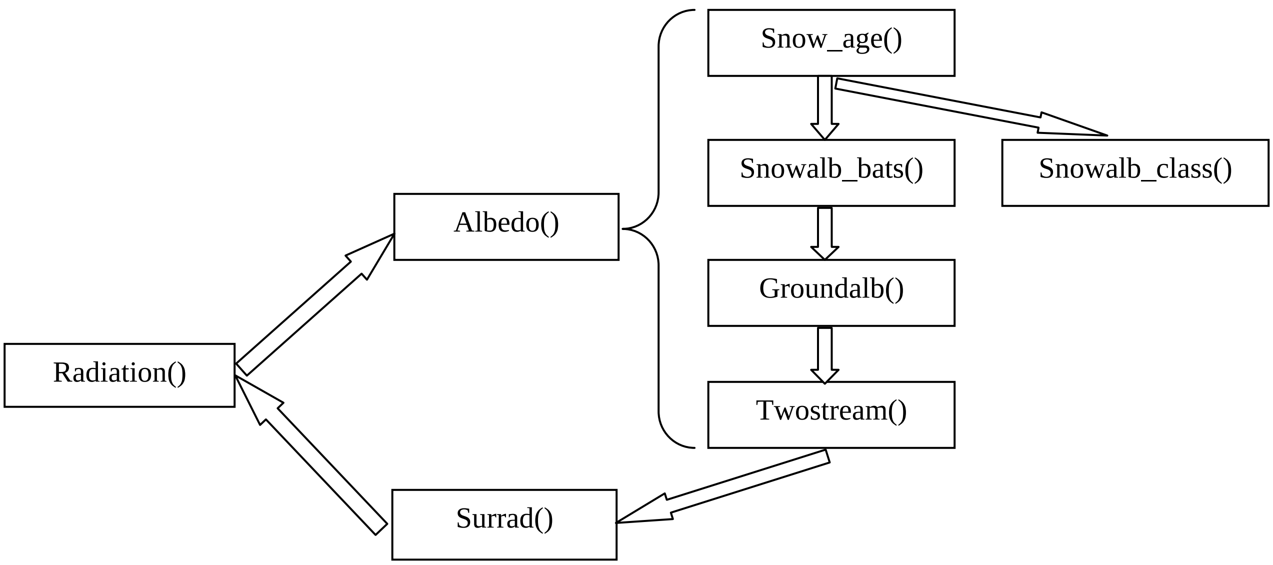


Figure 1 Flow chart of the radiation subroutine.

3.2 Snow Age

$$A_1 = e^{5\times10^3\left(\frac{1}{T_{frz}}-\frac{1}{T_g}\right)}$$

$$A_2 = e^{5\times10^4\left(\frac{1}{T_{frz}}-\frac{1}{T_g}\right)}$$

$$A_3 = 0.3$$

$$A_t = A_1 + A_2 + A_3$$

where A_1, A_2, A_3, and A_t represent the effects of grain growth due to vapor diffusion, the effects of grain growth at freezing of melt water, effects of soot, and the total effects of these three ones.

$$\tau_{ss} = \left(\tau_{ss} + 1\times10^{-6} dtA_t\right)\left(1 - S_w + S_m\right)$$

$$S_a = \frac{\tau_{ss}}{\tau_{ss} + 1}$$

Where S_a is snow age.

3.3 Snow albedo from bats

$$Z_c = \frac{1.5}{1 + 4\cos Z} - 0.5$$

$$\alpha_{sd1} = \alpha_{si1} + 0.4Z_c\left(1 - \alpha_{si1}\right)$$

$$\alpha_{sd2} = \alpha_{si2} + 0.4Z_c\left(1 - \alpha_{si2}\right)$$

$$\alpha_{si1} = 0.95(1 - 0.2A_c)$$

$$\alpha_{si2} = 0.65(1 - 0.5A_c)$$

3.4 Snow albedo from class

$$\alpha_1 = 0.55 + \left(\alpha_{old} - 0.55\right)e^{-0.01dt/3600}$$

$$\alpha = \alpha_1 + f_{sn}dt\left(0.84 - \alpha_1\right)$$

Where we make an assumption that the fresh snow density is 100kg/m3 and 1cm snow depth will fully cover the old snow.

$$\alpha_{sd1} = \alpha_{sd2} = \alpha_{si1} = \alpha_{si2} = \alpha$$

3.5 Ground albedo

$$C_w = 0.11 - 0.4W_s$$

where C_w is correction factor for soil albedo because of soil water which can change the soil albedo in a relative large range; W_s is volumetric soil water content.

For soil:

$$\alpha_d = \begin{cases} \alpha_{sat} + C_w & (\alpha_{sat} + C_w < \alpha_{dry}) \\ \alpha_{dry} & (\alpha_{sat} + C_w > \alpha_{dry}) \end{cases}$$

Based on the above equation we get that higher soil water content corresponds to lower soil albedo, which results from higher heat capability of water than that of soil and the transparent property of water.

$$\alpha_i = \alpha_d$$

For lake, when $T_g > T_{frz}$

$$\alpha_d = \frac{0.06}{(\cos Z)^{1.7}} + 0.15$$

$$\alpha_i = 0.06$$

when $T_g < T_{frz}$

$$\alpha_d = \alpha_{lake}$$

$$\alpha_i = \alpha_d$$

For desert and semi-desert,

$$\alpha_{dd} = \alpha_d + 0.1$$

$$\alpha_{id} = \alpha_i + 0.1$$

which are based on the fact that desert and semi-desert usually has a higher albedo.

So the ground albedo have the following forms:

$$\alpha_{gd} = \alpha_d (1 - f_{sn}) + \alpha_{dsn} f_{sn}$$

$$\alpha_{gi} = \alpha_i (1 - f_{sn}) + \alpha_{isn} f_{sn}$$

3.6 A Modified Two-Stream Radiation-Transfer Scheme

The two-stream radiation-transfer scheme used in the default version of VISA to calculate the radiation transfer through canopy is replaced by a modified version of the scheme by inducing the total canopy gap probability, P_c, which equals to the sum of the between-crown gap probability, P_{bc}, and the within-crown gap probability, P_{wc}:

$$P_{bc} = e^{-\rho_t \pi R^2 / \cos(\theta')}$$

$$P_{wc} = \left(1 - P_{bc}\right) e^{-0.5 F_a H_d / \cos\theta}$$

$$K_{open} = \int_0^{\pi/2} P_{bc} \sin\left(2\theta\right) d\theta$$

$$P_C = \begin{cases} 1 - f_{grn} \\ P_{bc} + P_{wc} \end{cases}$$

where K_{open} is gap fraction for diffuse light.

Flux absorbed by vegetation, f_{ab}:

$$f_{ab} = 1 - f_{re} - (1 - \alpha_d) f_{td} - \left(1 - \alpha_i\right) f_{ti}$$

Where α_d is direct albedo of underlying surface, α_i is the diffuse albedo of underlying surface, f_{re} is the reflected flux by vegetation and ground,

$$f_{re} = \begin{cases} \left(\dfrac{h_1}{\sigma} + h_2 + h_3\right)\left(1 - P_c\right) + \alpha_d P_c & \left(for\ direct\ beam\right) \\ \left(h_7 + h_8\right)\left(1 - K_{open}\right) + \alpha_i K_{open} & \left(for\ diffuse\ beam\right) \end{cases}$$

And f_{td} is downward direct flux below vegetation:

$$f_{td} = \begin{cases} S_2 & \left(for\ direct\ beam\right) \\ 0 & \left(for\ diffuse\ beam\right) \end{cases}$$

And f_{ti} is downward diffuse flux below vegetation:

$$f_{ti} = \begin{cases} \left(\dfrac{h_4 S_2}{\sigma} + h_5 S_1 + \dfrac{h_6}{S_1}\right)\left(1 - P_c\right) & \left(for\ direct\ beam\right) \\ \left(h_9 S_1 + h_{10} / S_1\right)\left(1 - K_{open}\right) + K_{open} & \left(for\ diffuse\ beam\right) \end{cases}$$

Where S_1 and S_2 are expressed as,

$$S_1 = e^{-H\ L}$$

$$S_2 = e^{-\tau_{dir} L}$$

Where L is one-sided leaf and stem area index (m2/m2), τ_{dir} is optical depth of direct beam per unit leaf area, and H can be calculated as following:

$$H = \frac{\sqrt{t_1}}{\tau_{avd}}$$

Where, τ_{avd} is average diffuse optical depth. In the above equations, $h_i (i = 1 \sim 10)$ are intermediate variables used to simplify the equations, and they are express as the following:

$$h_1 = -t_0 \Omega \beta_d (b - t_0) - \Omega^2 \beta_i t_0 (1 - \beta_d)$$

$$h_2 = \left[\frac{t_2 t_6}{S_1} - (b - \tau_{avd} H) t_7 \right] \Big/ d_1$$

$$h_3 = \left[t_3 t_6 S_1 - (b + \tau_{avd} H) t_7 \right] \Big/ d_1$$

$$h_4 = -t_0 \Omega (1 - \beta_d)(b + t_0) - \Omega^2 \beta_i \beta_d t_0$$

$$h_5 = \left(\frac{t_4 t_8}{S_1} + t_9 \right) \Big/ d_2$$

$$h_6 = (t_5 t_8 S_1 + t_9) / d_2$$

$$h_7 = \Omega \beta_i t_2 / (d_1 S_1)$$

$$h_8 = \Omega \beta_i t_3 S_1 / d_1$$

$$h_9 = t_4 / (d_2 S_1)$$

$$h_{10} = -t_5 S_1 / d_2$$

where, $t_i (i = 0 \sim 9)$, like h_i, are intermediate variables and expressed as the following,

$$t_0 = \tau_{avd} \tau_{dir}$$

$$t_1 = b^2 - c^2$$

$$t_2 = u_1 - \tau_{avd} H$$

$$t_3 = u_1 + \tau_{avd} H$$

$$t_4 = u_2 - \tau_{avd} H$$

$$t_5 = u_2 + \tau_{avd} H$$

$$t_6 = t_0 \Omega \beta_d - \frac{h_1 (b + t_0)}{\sigma}$$

$$t_7 = \left[t_0 \Omega \beta_d - \Omega \beta_i - \frac{h_1}{\sigma} (u_1 + t_0) \right] S_2$$

$$t_8 = \frac{h_4}{\sigma}$$

$$t_9 = \left[u_3 - t_8 (u_2 - t_0) \right] S_2$$

And is upscatter parameter for direct beam radiation, is upscatter parameter for diffuse radiation

And is fraction of intercepted radiation that is scattered and express as,

$$\Omega = \begin{cases} \Omega_L & (t > t_{frz}) \\ (1 - f_{wet}) \Omega_L + f_{wet} \Omega_S & (t \leq t_{frz}) \end{cases}$$

Also b, c, d_1 and d_2 are intermediate variables and expressed as the following,

$$b = 1 - \Omega + \Omega \beta_i$$

$$c = \Omega \beta_i$$

$$d_1 = \frac{(b + \tau_{avd}) t_2}{S_1} - (b - \tau_{avd} H) t_3 S_1$$

$$d_2 = \frac{t_4}{S_1} - t_5 S_1$$

Where u_1, u_2 and u_3 are expressed as,

$$u_1 = b - \frac{c}{\alpha}$$

$$u_2 = b - c\alpha$$

$$u_3 = t_0 \Omega (1 - \beta_d) + c\alpha$$

Where α is albedo of underlying surface,

$$\alpha = \begin{cases} \alpha_d & (for\ \ direct\ \ beam) \\ \alpha_i & (for\ \ diffuse\ \ beam) \end{cases}$$

and β_i and β_d are upscatter parameter for diffuse radiation and upscatter parameter for direct beam radiation,

$$\beta_d = \begin{cases} \beta_{dl} & (t > t_{frz}) \\ (1 - f_{wet})\Omega_L \beta_{dl} + f_{wet}\Omega_S \beta_{ds} & (t \le t_{frz}) \end{cases}$$

$$\beta_i = \begin{cases} \beta_{di} & (t > t_{frz}) \\ (1 - f_{wet})\Omega_L \beta_{il} + f_{wet}\Omega_S \beta_{is} & (t \le t_{frz}) \end{cases}$$

Where A is single scattering albedo, and expressed as, And β_{dl} and β_{il} are the same as β_d and β_i, but for leaves, and f_{wet} is the fraction of leaf and stem area index that is wetted, Ω_L and Ω_S are fraction of intercepted radiation that is scattered by leaves and stem, respectively.

$$\beta_{dl} = \left(1 + \frac{\tau_{avd}}{\tau_{dir}}\right) A \Big/ \left(\Omega_L \tau_{avd} \tau_{dir}\right)$$

$$\beta_{il} = \rho \left(1 + \frac{X_L}{2}\right)^2 \Big/ \Omega_L$$

Where A is single scattering albedo, and expressed as,

$$A = 0.5(\rho + \tau) \frac{\varphi_1 + \varphi_2 \cos\theta'}{\varphi_1 + 2\varphi_2 \cos\theta'} \left(1 - \frac{\varphi_1 \cos\theta'}{(\varphi_1 + 2\varphi_2 \cos\theta')} \log \frac{\varphi_1 (1 + \cos\theta') + 2\varphi_2 \cos\theta'}{\varphi_1 \cos\theta'}\right) \Big/ (\varphi_1 + 2\varphi_2 \cos\theta')$$

$$\Omega_L = \rho + \tau$$

$$\tau_{avd} = \left(1 - \frac{\varphi_1}{\varphi_2} \log \frac{\varphi_1 + \varphi_2}{\varphi_1}\right) \Big/ \varphi_2$$

$$\tau_{dir} = \frac{\varphi_1}{\cos\theta'} + \varphi_2$$

$$\varphi_1 = 0.5 - 0.633 X_L - 0.33 X_L{}^2$$

$$\varphi_2 = 0.877(1 - 2\varphi_1)$$

$$x_L = \begin{cases} x & (x>0.01) \\ 0.01 & (x\le 0.01) \end{cases}$$

where, x ranges from -0.4 to 0.6.

$$\theta' = \tan^{-1}\left(\frac{H_{top} - H_{bot}}{2R}\tan(\theta)\right)$$

where, H_{top} and H_{bot} are the top and bottom heights of the crown, R is the horizontal crown radius.

$$\theta = \begin{cases} Z & (Z<89.5^\circ) \\ 89.5^\circ & (Z\ge 89.5^\circ) \end{cases}$$

where, Z is zenith angle.

3.7 Surface radiation initialization

$$f_{sha} = 1 - f_{sun}$$

$$LAI_{sun} = ELAI \times f_{sun}$$

$$LAI_{sha} = ELAI \times f_{sha}$$

$$VAI = ELAI + ESAI$$

If VAI is greater than zero, the land is considered as covered by the canopy, or else no canopy in the grid.

3.8 Surface radiation

The following equations are used to calculate the solar radiation absorbed by vegetation.

$$C_{ad} = S_d f_{abd}$$

$$C_{ai} = S_i f_{abi}$$

$$S_{av} = C_{ad} + C_{ai}$$

$$F_{sa} = C_{ad} + C_{ai}$$

These two equations represent the transmitted solar fluxes incident on ground.

$$T_d = S_d F_{dd}$$

$$T_i = S_d F_{id} + S_i F_{ii}$$

Solar radiation absorbed by ground surface is represented as:

$$R_a = T_d\left(1-\alpha\right)+T_i\left(1-\alpha\right)$$

$$R_g = R_a$$

$$F_{sa} = C_{ad} + C_{ai} + R_a$$

Partition visible canopy absorption to sunlit and shaded fractions to get average absorbed parameter for sunlit and shaded leaves.

$$f_{LAI} = \frac{LAI}{LAI + SAI}$$

$$P_{sun} = \left(C_{ad} + C_{ai}\right) f_{LAI} \big/ LAI_{sun}$$

$$P_{sha} = C_{ai}\, f_{sha} \big/ LAI_{sha}$$

Here are reflected solar radiations.

$$R_{nir} = \alpha_{dnir} R_{dnir} + \alpha_{inir} R_{inir}$$

$$R_{vis} = \alpha_{dvis} R_{dvis} + \alpha_{ivis} R_{ivis}$$

$$F_{sr} = R_{nir} + R_{vis}$$

4. Momentum, Sensible Heat, and Latent Heat Fluxes

4.1 Momentum, Sensible Heat, and Latent Heat Fluxes for Bare Ground

4.1.1 Overall

$$c_{ir} = \varepsilon_g \sigma \tag{4-1}$$

Where c_{ir} coefficients for longwave radiation as function of T_i^4.

$$c_{st} = 2k_{so/sn} / \Delta z \tag{4-2}$$

Where c_{st} coefficients for ground heat flux as function of T_i.

$$z_{0h} = \begin{cases} z_{0m} & \text{1st iteration} \\ z_{0m} \cdot e^{-C_{zil} k \left(1/\sqrt{\upsilon}\right)\sqrt{u_* z_{0m}}} & \text{other iterations} \end{cases} \tag{4-3}$$

Where z_{0m} is roughness length for momentum (m), which is the theoretical height at which wind speed is zero [*Bonan*, 2008, *Ecological Climatology*, P208]. z_{0h} is roughness lenth for heat flux (m).

$$r_{aM} = \max\left(1, \frac{1}{C_m \cdot u_*}\right) \tag{4-4}$$

Where r_M is the aerodynamic resistance for momentum. C_m is momentum drag coefficient.

$$r_{aH} = \max\left(1, \frac{1}{C_h \cdot u_*}\right) \tag{4-5}$$

Where r_{aH} is the aerodynamic resistance for sensible heat. C_h is sensible heat exchange coefficient.

$$r_{aW} = r_{aH} \tag{4-6}$$

Where r_{aW} is the aerodynamic resistance for momentum. The drag coefficient for momentum and sensible heat can be calculated using Monin-Obukhov similarity theory or the method used in original Noah LSM introduced by *Chen* [1997], which are introduced in Section 4.1.2 and Section 4.1.3 .

Saturation vapor pressure at ground temperature (pa), e_s:

$$e_s = \begin{cases} e_{sw} & T_g > 0 \\ e_{si} & T_g \le 0 \end{cases} \tag{4-7}$$

Where T_g is ground temperature in °C. e_{sw} and e_{si} are saturation vapor pressure for water and ice (Pa), both of which are calculated in Section ???.

The derivative of saturation vapor pressure as a function of ground temperature (Pa K^{-1}):

$$\frac{d(e_s)}{dt} = \begin{cases} \frac{d(e_{sw})}{dt} & T_g > 0 \\ \frac{d(e_{si})}{dt} & T_g \le 0 \end{cases} \tag{4-8}$$

Where both of $d(e_{sw})/dt$ and $d(e_{si})/dt$ are also calculated in Section ???.

Coefficients for sensible heat as function of surface temperature, c_{sh}:

$$c_{sh} = \frac{\rho_{air} C_{air}}{r_{aH}} \tag{4-9}$$

Coefficients for evaporation as function of $e_s(T_i)$, c_{ev}:

$$c_{ev} = \frac{\rho_{air} C_{air}}{r_s r_{aH} \gamma} \tag{4-10}$$

Net longwave radiation (W m^{-2}) (positive towards the atmosphere) at the surface:

$$L_n = -L_{atm} \downarrow + L \uparrow \tag{4-11}$$

Where $L_{atm}\downarrow$ is the downward longwave radiation (W m^{-2}). $L\uparrow$ is the upward longwave radiation (W m^{-2}).

For non-vegetated surfaces:

$$L_n' = -\alpha_g L_{atm} \downarrow + \varepsilon_g \sigma T_g^4 \tag{4-12}$$

Where L_n' is preliminary net longwave radiation (W m^{-2}), α_g is the ground absorptivity. ε_g is the ground emissivity. T_g is the ground temperature (K).

$$H' = c_{sh}\left(T_g - \theta_a\right) \tag{4-13}$$

Where H' is the preliminary sensible heat flux (W m^{-2}). θ_a is potential temperature (K).

$$E' = c_{ev}\left(e_s RH - e_a\right) \tag{4-14}$$

Where E' is the preliminary water vapor flux (W m^{-2}). e_a is vapor pressure of air (Pa). RH is relative humidity.

$$G' = c_{gh}\left(T_g - T_{i+1}\right) \tag{4-15}$$

Where G' is the preliminary ground heat flux (W m^{-2}). c_{gh} is Coefficients for ground heat flux as a function of surface temperature.

$$\Delta T_g = \frac{\vec{S}_g - L_n - H - E - G}{4\varepsilon_g \sigma T_g^3 + c_{sh} + c_{ev} + c_{gh}} \tag{4-16}$$

Where ΔT is the ground temperature change (K). $\vec{S}_g$ is the solar radiation absorbed by ground (W m^{-2}).

Update L_n, H, E, G, and T_g.

$$L_n = L_n' + 4\varepsilon_g \sigma T_g^3 \Delta T_g = -\alpha_g L_{atm}\downarrow + \varepsilon_g \sigma T_g^4 + 4\varepsilon_g \sigma T_g^3 \Delta T_g \quad (4\text{-}17)$$

$$H = H' + c_{sh}\Delta T_g = c_{sh}\left(T_g + \Delta T_g - \theta_a\right) \quad (4\text{-}18)$$

$$E = E' + c_{ev\,g} = c_{ev}\left(e_s RH - e_a + \frac{d(e_s)}{dt}\Delta T\right) \quad (4\text{-}19)$$

$$G = G' + c_{gh}\Delta T = c_{gh}\left(T_g + \Delta T - T_{i+1}\right) \quad (4\text{-}20)$$

$$T_g' = T_g + \Delta T_g \quad (4\text{-}21)$$

Where T_g' is the updated ground temperature (K).

If there is snow on the ground and $T_g' > T_f$, reset T_g' equal to T_f.

$$T_g' = T_f \quad (4\text{-}22)$$

Then reevaluate ground fluxes, the L_n, H, E are recalculated using the equations (1-13) ~ (1-14). The ground heat flux is recalculated using these new fluxes.

$$G = \vec{L}_g - \left(L_n + H + E\right) \quad (4\text{-}23)$$

Wind stresses.

$$\tau_{wind,x} = \rho_{air} C_m u_* u \quad (4\text{-}24)$$

Where $\tau_{wind,x}$ is the wind stress in E-W direction (N m^{-2}).

$$\tau_{wind,y} = \rho_{air} C_m u_* v \quad (4\text{-}25)$$

Where $\tau_{wind,y}$ is the wind stress in N-S direction (N m^{-2}).

4.1.2 Monin-Obukhov

Before proceeding to further calculation, it has to make sure reference height (z_a) is greater than zero plane displacement (d).

$$TMPCM = \ln\frac{z_a - d}{z_{0m}} \quad (4\text{-}26)$$

$$TMPCH = \ln \frac{z_a - d}{z_{0h}} \tag{4-27}$$

$$T_{v,a} = T_a (1 + 0.61 q_a) \tag{4-28}$$

Where $T_{v,a}$ is the temporary virtual temperature (K). T_a is the air temperature at reference height (K). q_a is the specific humidity (kg kg^{-1}).

$$L = \frac{-u_*^3}{\kappa \frac{g}{T_{v,a}} \frac{H}{\rho_{air} c_{air}}} \tag{4-29}$$

Where L is the Monin-Obukhov length (m). κ is Von Karman constant, and $\kappa = 0.4$. g is the acceleration of gravity (m s^{-2}), and $g = 9.80616$ m s^{-2}.

$$\zeta = \min\left(\frac{z_a - d}{L}, 1 \right) \tag{4-30}$$

Where ζ is the Monin-Obukhov turbulent stability parameter.

$$\Psi_m' = \begin{cases} 2\ln\left(\frac{1+x}{2}\right) + \ln\left(\frac{1+x^2}{2}\right) - 2\arctan x + \frac{\pi}{2}, & \zeta < 0 \\ -5\zeta, & 0 \le \zeta \le 1 \end{cases} \tag{4-31}$$

$$x = (1 - 16\zeta)^{1/4} \tag{4-32}$$

$$\Psi_h' = \begin{cases} 2\ln\left(\frac{1+x^2}{2}\right), & \zeta < 0 \\ -5\zeta, & 0 \le \zeta \le 1 \end{cases} \tag{4-33}$$

$$\Psi_m = \begin{cases} \Psi_m', & \text{for the } 1^{st} \text{ } iteration \\ \frac{(\Psi_m + \Psi_m')}{2}, & \text{for other } iterations \end{cases} \tag{4-34}$$

$$\Psi_h = \begin{cases} \Psi_h', & \text{for } iteration \text{ } 1 \\ \frac{(\Psi_h + \Psi_h')}{2}, & \text{for other } iterations \end{cases} \tag{4-35}$$

Where Ψ_m is the momentum stability correction, weighted by prior iterations. Ψ_h is the sensible heat stability correction, weighted by prior iterations.

$$C_m = \begin{cases} \dfrac{\kappa^2}{\left(\ln\dfrac{z_a - d}{z_{0m}} - \Psi_m\right)^2} & \text{if } \ln\dfrac{z_a - d}{z_{0m}} - \Psi_m > 10^{-6} \\ \dfrac{\kappa^2}{\left(10^{-6}\right)^2}, & \text{if } \ln\dfrac{z_a - d}{z_{0m}} - \Psi_m \le 10^{-6} \end{cases} \tag{4-36}$$

$$C_h = \begin{cases} \dfrac{\kappa^2}{\left(\ln\dfrac{z_a - d}{z_{0h}} - \Psi_h\right)^2} & \text{if } \ln\dfrac{z_a - d}{z_{0h}} - \Psi_h > 10^{-6} \\ \dfrac{\kappa^2}{\left(10^{-6}\right)^2}, & \text{if } \ln\dfrac{z_a - d}{z_{0h}} - \Psi_h \le 10^{-6} \end{cases} \tag{4-37}$$

4.1.3 Chen's Implementation in Original Noah LSM

The method used in this part was introduced by *Chen* [1997].

4.1.3.1 Initial Iteration (1st Iteration)

Here we use superscript "$\bar{j}$" to indicate iteration. For the 1st iteration, the friction velocity u_* is adjusted using Beljars correction [*Beljaars and Viterbo*, 1998].

$$w_*^2 = \begin{cases} 0 & \text{if } C_h\left(\theta_a - T_g\right) = 0 \\ 1.44 \cdot \left|\beta g h_{PBL} C_h\left(\theta_a - T_g\right)\right|^{2/3} & \text{if } C_h\left(\theta_a - T_g\right) \ne 0 \end{cases} \tag{4-38}$$

$$u_* = \begin{cases} \sqrt{C_m \cdot u} \ge 0.07 & \text{if } C_h\left(\theta_a - T_g\right) = 0 \\ \sqrt{C_m \cdot \sqrt{u^2 + w_*^2}} \ge 0.07 & \text{if } C_h\left(\theta_a - T_g\right) \ne 0 \end{cases} \tag{4-39}$$

Where w_* is friction velocity in vertical direction (m s^{-1}), u_* is friction velocity (m s^{-1}), u is wind speed and $u^2 \ge 10^{-4}$, $\beta = \dfrac{1}{270}$, g is gravity acceleration, h_{PBL} is the planetary boundary layer depth and h_{PBL} =1000m, θ_a is potential temperature at reference height (k), and T_g is ground surface temperature.

$$\zeta^{\bar{j}} = \frac{k\beta g C_H\left(\theta_a - T_g\right)}{u_*^3} \tag{4-40}$$

4.1.3.2 Other Iterations

For later iterations,

$$z_{0h} = z_{0m} \cdot \exp\left(-kC_{zil}\sqrt{R_e^*}\right) \tag{4-41}$$

Where z_{0h} is roughness for heat, and $z_{0h} > 10^{-6}$.

$$R_e^* = \frac{u_* z_{0,m}}{\upsilon} \tag{4-42}$$

$$\bar{\zeta}^{\,j} = \begin{cases} \bar{\zeta}^{\,j-1} & \text{for } z_a + z_{0h} > z_{oh,min} \\ \dfrac{z_{oh,min}}{z_a + z_{0h}} & \text{for } z_a + z_{0h} \le z_{oh,min} \end{cases} \tag{4-43}$$

Beljaars correction for friction velocity u_*.

$$u_* = \sqrt{C_m \cdot \sqrt{u^2 + w_*^2}} \ge 0.07 \tag{4-44}$$

$$C_m = \max\left(\frac{k u_*}{\Psi_m(y_m) - \Psi_m(x_m) + \ln(z_a + z_{0m}) - \ln z_{0h}}, \frac{0.001}{z_a} \right) \tag{4-45}$$

$$C_h = \max\left(\frac{k u_*}{\Psi_h(y_h) - \Psi_h(x_h) + \ln(z_a + z_{0h}) - \ln z_{0h}}, \frac{0.001}{z_a} \right) \tag{4-46}$$

Where we can use either Łobocki's or Paulson's surface functions to compte stability correction for momentum (Ψ_m) and stability correction for sensible heat flux (Ψ_h).

1) Paulson's surface functions [*Paulson*, 1970].

$$\Psi_m = \begin{cases} -5\zeta & 0 < \zeta < 1 \\ 2\ln\left(\dfrac{1+x}{2}\right) + \ln\left(\dfrac{1+x^2}{2}\right) - 2\arctan(x) + \dfrac{\pi}{2} & -5 < \zeta < 0 \end{cases} \tag{4-47}$$

$$\Psi_h = \begin{cases} -5\zeta & 0 < \zeta < 1 \\ 2\ln\left(\dfrac{1+x^2}{2}\right) & -5 < \zeta < 0 \end{cases} \tag{4-48}$$

Where $\zeta = z/L$ and $x = (1\text{-}16\zeta)^{1/4}$.

$$x_m = \begin{cases} [1 - 16 z_{0m}\zeta]^{1/4} & \zeta < 0 \\ z_{0m}\zeta & \zeta \ge 0 \end{cases} \tag{4-49}$$

$$y_m = \begin{cases} [1 - 16(z_a + z_{0m})]^{1/4} & \zeta < 0 \\ (z_a + z_{0m})\zeta \le z_{0h,\min} & \zeta \ge 0 \end{cases} \tag{4-50}$$

Where

$$x_h = \begin{cases} [1 - 16 z_{oh}\zeta]^{1/4} & \zeta < 0 \\ z_{0h}\zeta & \zeta \ge 0 \end{cases} \tag{4-51}$$

$$y_h = \begin{cases} \left[1-16\left(z_a+z_{0h}\right)\right]^{1/4} \geq \left[1-16 z_{0h,\min}\right]^{1/4} & \zeta < 0 \\ z_{0h,\min} \leq \left(z_a+z_{0h}\right)\zeta \leq z_{0h,\max} & \zeta \geq 0 \end{cases} \tag{4-52}$$

2) Łobocki's surface functions [*Łobocki*, 1993].

$$\Psi_m = \begin{cases} \zeta R_{FC}^{-1} - 2.076\left[1-\left(\zeta+1\right)^{-1}\right] & 0 \leq \zeta < 1 \\ -0.96\ln\left(1-4.5\zeta\right) & -5 < \zeta < 0 \end{cases} \tag{4-53}$$

$$\Psi_h = \begin{cases} \dfrac{\zeta R_{ic}}{R_{FC}^2 \phi_T(0)} - 2.076\left[1-\exp\left(-1.2\zeta\right)\right] & 0 \leq \zeta < 1 \\ -0.96\ln\left(1-4.5\zeta\right) & -5 < \zeta < 0 \end{cases} \tag{4-54}$$

Where

$$R_{FC} = \frac{B_1 - 6A_1}{B_1 + 12A_1 + 3B_2} = 0.191 \tag{4-55}$$

$$R_{ic} = \frac{B_1}{3A_2} \frac{\left(\gamma_1 - C_1\right)\gamma_2 - \left(6A_1 + 3A_2\right)\gamma_1 / B_1}{\gamma_1 + \gamma_2} = 0.183 \tag{4-56}$$

R_{FC} is the critical flux Richardson number, R_{ic} is the critical gradient Richardson number (but I got 0.195 instead of 0.183 when I used the values in [Łobocki, 1993]), and $\phi_T(0)=0.8$ is the dimensionless velocity gradient for neutral conditions. The constants used in the above computation are from *Mellor and Yamada* [1982]; they are,

$$(A_1, A_2, B_1, B_2, C_1) = (0.92, 0.74, 16.6, 10.1, 0.08)$$

$$x_m = z_{0m}\zeta \tag{4-57}$$

$$y_m = \begin{cases} \left(z_a + z_{0m}\right)\zeta & \zeta < 0 \text{ and } \left(z_a + z_{0m}\right)\zeta < z_{0h,\max}, \\ z_{0h,\max} & \zeta \geq 0 \text{ and } \left(z_a + z_{0m}\right)\zeta > z_{0h,\max}, \end{cases} \tag{4-58}$$

$$x_h = z_{0h}\zeta \tag{4-59}$$

$$y_m = \begin{cases} \left(z_a + z_{0h}\right)\zeta & \zeta < 0 \text{ and } \left(z_a + z_{0h}\right)\zeta < z_{0h,\max}, \\ z_{0h,\max} & \zeta \geq 0 \text{ and } \left(z_a + z_{0h}\right)\zeta > z_{0h,\max}, \end{cases} \tag{4-60}$$

Zilitinkevitch fix for z_{oh}.

$$z_{0h} = z_0 \cdot \exp\left(-kC_{zil}\left(\sqrt{\nu}\right)^{-1}\sqrt{u_* \cdot z_0}\right) \geq 10^{-6} \tag{4-61}$$

An IF statement to avoid tangent linear problems near 0.

$$w_*^2 = \begin{cases} 0 & \text{if } C_H\left(\theta_a - T_g\right) = 0 \\ 1.44 \cdot \left|\beta g h_{PBL} C_H\left(\theta_a - T_g\right)\right|^{2/3} & \text{if } C_H\left(\theta_a - T_g\right) \neq 0 \end{cases} \tag{4-62}$$

$$\zeta^{\bar{j}}{}' = \frac{k\beta g C_H\left(\theta_a - T_g\right)}{u_*^3} \tag{4-63}$$

Update the Monin-Obukhov stability parameter using a weight of ω for the previous and 1–ω for the current.

$$\zeta^{\bar{j}} = \zeta^{\bar{j}} \cdot \omega + \zeta^{\overline{j\text{-}1}} \cdot \left(1 - \omega\right) \tag{4-64}$$

Where ω=0.15.

4.2 Momentum, Sensible Heat, and Latent Heat Fluxes for Vegetated Ground

4.2.1 Canopy fluxes

The effective VAI (Vegetation Area Index, i.e. LAI+SAI) is converted from grid-based VAI：

$$VAI_e = \frac{VAI}{f_{veg}}$$

where f_{veg} is greenness vegetation fraction.

Similarly, the sunlit and shaded LAI (one-sided) are converted

$$LAI_{sun,e} = \frac{LAI_{sun}}{f_{veg}}$$

$$LAI_{shd,e} = \frac{LAI_{shd}}{f_{veg}}.$$

All the VAI_e, $LAI_{sun,e}$, $LAI_{shd,e}$ are limited to be less than or equal to 6.

Wind speed at the top of canopy layer is

$$u_c = u_r \bullet \frac{\ln\left(h_{can}/z_{0M}\right)}{\ln\left(z_{lvl}/z_{0M}\right)}$$

where u_r is wind speed at the reference level, h_{can} is canopy height, z_{lvl} is the height of reference level, and z_{0M} is roughness length for momentum.

The fluxes between canopy and the atmosphere, and vegetation temperature are calculated by "stability iterations".

$$z_{0H} = z_{0M}$$

$$z_{0H,g} = z_{0M,g}$$

Calculate momentum drag coefficient C_M and sensible heat exchange coefficient C_H either from M-O scheme or from Chen97 scheme.

Aerodynamic resistance for momentum over canopy is

$r_{aM,c} = \max(1/(C_M u_r), 1)$, and the resistance for sensible heat and water vapor are

$r_{aH,c} = r_{aW,c} = \max(1/(C_H u_r), 1)$

$r_{aM,g}$, $r_{aH,g}$, $r_{aW,g}$ are calculated by RAGRB subroutine.

At the 1st iteration step, determine the rate of sunlit leaf photosynthesis A_{sun} and shaded leaf photosynthesis A_{shd} by either Bell-Berry scheme or Jarvis scheme.

To prepare for the sensible heat flux between canopy and atmosphere, below items are calculated:

sensible heat conductance from canopy air to air at reference height $C_{aH} = 1/r_{aH,c}$,

sensible heat conductance, from leaf surface to canopy air $C_{vH} = 2VAI_e/r_b$

??? conductance $C_{gH} = 1/r_{aH,g}$

$$ATA = \frac{T_{h,air} \bullet C_{aH} + T_g \bullet C_{gH}}{C_{aH} + C_{vH} + C_{gH}}$$

$$BTA = \frac{C_{vH}}{C_{aH} + C_{vH} + C_{gH}}$$

$$CSH = (1 - BTA) \bullet \rho_{air} \bullet C_{p,air} \bullet C_{vH}$$

where $T_{h,air}$ is the potential temperature at reference level, T_g is ground temperature.

To prepare for the latent heat flux between canopy and atmosphere, below items are calculated:

latent heat conductance from canopy air to air at reference height $C_{aW} = 1/r_{aW,c}$

evaporation conductance, leaf to canopy air $C_{eW} = f_{wet} \bullet VAI_e / r_b$, where f_{wet} is the wet fraction of canopy.

transpiration conductance, leaf to canopy air $C_{tW} = (1 - f_{wet}) \bullet LAI_{sun,e} / (r_b + r_{s,sun})$

latent heat conductance, ground to canopy air $C_{gW} = 1 / (r_{aW,g} + r_{surf})$

$$AEA = \frac{e_{air} \bullet C_{aW} + e_{sat,g} \bullet C_{gW}}{C_{aW} + C_{eW} + C_{tW} + C_{gW}}$$

$$BEA = \frac{C_{eW} + C_{tW}}{C_{aW} + C_{eW} + C_{tW} + C_{gW}}$$

$$CEV = (1 - BEA) \bullet C_{eW} \bullet \rho_{air} \bullet C_{p,air} / \gamma$$

$$CTR = (1 - BEA) \bullet C_{tW} \bullet \rho_{air} \bullet C_{p,air} / \gamma$$

where γ is psychrometric constant.

Then evaluate the canopy surface fluxes with current temperature and solve vegetation temperature.

Canopy air temperature $T_{a,H} = ATA + BTA \bullet T_v$

canopy air water vapor pressure $e_{a,H} = AEA + BEA \bullet e_{sat,v}$

net longwave radiation

$$L_{a,v} = f_{veg} \left\{ -\varepsilon_v \bullet \left[1 + (1 - \varepsilon_v)(1 - \varepsilon_g) \right] \bullet L_{air}^{\downarrow} - \varepsilon_v \varepsilon_g \sigma T_g^4 + \left[2 - \varepsilon_v (1 - \varepsilon_g) \right] \bullet \varepsilon_v \sigma T_v^4 \right\}$$

where ε_v and ε_g are vegetation and ground emissivity, respectively, $L_{air}^{\downarrow}$ is atmospheric longwave radiation, σ is Stefan-Boltzmann constant.

sensible heat flux $H_v = f_{veg} \bullet \rho_{air} \bullet C_{p,air} \bullet C_{vH} \bullet (T_v - T_{a,H})$

evaporation heat flux $EV_v = f_{veg} \bullet \rho_{air} \bullet C_{p,air} \bullet C_{eW} \bullet (e_{sat,v} - e_{a,H}) / \gamma$

$$EV_v = \begin{cases} EV_v; & EV_v < \lambda \bullet Itc_c / dt \\ \lambda \bullet Itc_c / dt; & EV_v \geq \lambda \bullet Itc_c / dt \end{cases}$$ where Itc_c is intercepted liquid water by canopy.

transpiration heat flux $TR_v = f_{veg} \bullet \rho_{air} \bullet C_{p,air} \bullet C_{tW} \bullet \left(e_{sat,v} - e_{a,H}\right) / \gamma$,

Change in T_v is

$$dT_v = \frac{S_{a,v} - L_{a,v} - H_v - EV_v - TR_v}{f_{veg} \bullet \left(4\left[2 - \varepsilon_v \left(1 - \varepsilon_g\right)\right] \bullet \varepsilon_v \sigma T_v^3 + CSH + \left(CEV + CTR\right) \bullet \frac{de_s}{dt} \right)}$$

Finally update T_v by $T_v = T_v + dT_v$. Then repeat the above iteration procedure until a user-defined number of iteration is reached.

The next step is the iteration to compute under-canopy fluxes and ground temperature.

Similarly, we use the stability iteration,

First, calculate the heat fluxes,

Longwave radiation flux: $L_{ag,v} = \varepsilon_g \sigma T_{g,v}^4 + \left(-\varepsilon_g \left(1 - \varepsilon_v\right) L_{air}^{\downarrow} - \varepsilon_g \varepsilon_v \sigma T_v^4\right)$

Sensible heat flux: $H_{g,v} = \rho_{air} C_{p,air} \left(T_{g,v} - T_{a,H}\right) / r_{aH,g}$

Evaporation heat flux: $EV_{g,v} = \rho_{air} C_{p,air} \left(e_{sat,gv} Rh_{surf} - e_{a,H}\right) / \left[\gamma \left(r_{aW,g} - r_{surf}\right)\right]$

where Rh_{surf} is the relative humidity in surface snow/soil air space.

Ground heat flux $G_v = 2 \frac{\lambda_{isno+1}}{\Delta z_{isno+1}} \left(T_{g,v} - T_{isno+1}\right)$

where λ_{isno+1} is the thermal conductivity of the surface layer of snow or soil, Δz_{isno+1} is the layer thickness of the surface layer of snow or soil, and T_{isno+1} is the temperature of the surface layer.

The change of ground temperature under canopy

$$dT_{g,v} = \frac{S_{a\ gv} - L_{a\ gv} - H_{g,v} - EV_{g,v} - G_v}{4\varepsilon_g \sigma T_{g,v}^3 + \frac{\rho_{air} C_{p,air}}{r_{aH,g}} + \frac{\rho_{air} C_{p,air}}{\gamma\left(r_{aW,g} + r_{surf}\right)} \bullet \frac{de_s}{dt} + 2\frac{\lambda_{isno+1}}{\Delta z_{isno+1}}}$$

Then update all the heat fluxes according to $dT_{g,v}$

$$L_{a\ gv} = L_{a\ gv} + 4\varepsilon_g \sigma T_{g,v}^3 dT_{g,v}$$

$$H_{g,v} = H_{g,v} + \rho_{air} C_{p,air} dT_{g,v} \big/ r_{aH,g}$$

$$EV_{g,v} = EV_{g,v} + \rho_{air} C_{p,air} dT_{g,v} \left(de_s / dt\right) \Big/ \left[\gamma\left(r_{aW,g} - r_{surf}\right)\right]$$

$$G_v = G_v + 2\frac{\lambda_{isno+1}}{\Delta z_{isno+1}} dT_{g,v}$$

Repeat the above procedure until a user-defined iteration number is reached.

When OPT_STC=1, i.e. semi-implicit snow/soil temperature scheme is used, if $h_{snow} > 0.05$ and $T_{g,v} > T_{frz}$ (freezing point temperature), then set $T_{g,v} = T_{frz}$ and reevaluate all the flux as above, except that

$$G_v = S_{a\ gv} - L_{a\ gv} - H_{g,v} - EV_{g,v}$$

The wind stresses over vegetated ground:

$$\tau_{x,v} = -\rho_{air} \bullet C_M \bullet u_r \bullet U$$

$$\tau_{y,v} = -\rho_{air} \bullet C_M \bullet u_r \bullet V$$

and finally output 2-m temperature

$$T_{2m,v} = T_{a,H} - \frac{H_{g,v} + H_v}{\rho_{air} C_{p,air} u_*} \bullet \frac{1}{\kappa} \ln\left(\frac{2 + z_{0H}}{z_{0H}}\right)$$

where $\kappa = 0.4$ is von Karman constant, and u_* is friction velocity.

4.2.2 Bell-Berry stomatal conductance scheme

$$CF = \frac{p_{sfc} \bullet 10^6}{8.314 T_{sfc}}$$

leaf stomatal resistance $r_s = CF / BP$

Initialize leaf photosynthesis $PSN = 0$

Calculate several intermediate terms

$$Fnf = \min\left\{ N_{leaf} / N_{leaf,max}, 1 \right\}$$

$$T_c = T_v - T_{frz}$$

$$PPF = 4.6 APAR$$

$$J = PPF \bullet QE_{25}$$

$$KC = KC_{25} \bullet AKC^{(T_c - 25)/10}$$

$$KO = KO_{25} \bullet AKO^{(T_c - 25)/10}$$

$$AWC = KC \bullet \frac{1 + O2}{KO}$$

$$CP = 0.5 \frac{KC}{KO} \bullet O2 \bullet 0.21$$

$$V_{c,\max} = V_{c,\max,25} \bullet Fnf \bullet \beta_{tran} \bullet \frac{AV_{c,\max}^{(T_c - 25)/10}}{1 + \exp\left\{ \frac{\left[-2.2 \bullet 10^5 + 710 \bullet \left(T_c + 273.16 \right) \right]}{8.314 \bullet \left(T_c + 273.16 \right)} \right\}}$$

Calculate a first guess of CI

$$CI = 0.7CO_2 \bullet C3PSN + 0.4CO_2 \bullet (1 - C3PSN)$$

$$r_{lb} = \frac{r_b}{CF}$$

$$CEA = \max\left\{0.25EI \bullet C3PSN + 0.4EI \bullet (1 - C3PSN),\ \min\{EA, EI\}\right\}$$

Given the iteration number (currently set to be 3), calculate several variables through iterations

$$WJ = \frac{\max(CI - CP, 0) \bullet J}{(CI + 2CP) \bullet C3PSN} + J \bullet (1 - C3PSN)$$

$$WC = \frac{\max(CI - CP, 0) \bullet V_{c,\max}}{(CI + AWC) \bullet C3PSN} + V_{c,\max} \bullet (1 - C3PSN)$$

$$WE = 0.5V_{c,\max} \bullet C3PSN + \frac{4000 \bullet V_{c,\max} \bullet CI}{p_{sfc}(1 - C3PSN)}$$

Then update $PSN = IGS \bullet \min\{WJ, WC, WE\}$

$$CS = CO_2 - 1.37 r_{lb} p_{sfc} PSN$$

To solve Q

$$A = \frac{MP \bullet PSN \bullet p_{sfc} \bullet CEA}{CS \bullet EI + BP}$$

$$B = \left(\frac{MP \bullet PSN \bullet p_{sfc}}{CS} + BP\right) \bullet r_{lb} - 1$$

$$C = -r_{lb}$$

then $$Q = \begin{cases} -\dfrac{B + \sqrt{B^2 - 4AC}}{2} & B \geq 0 \\ -\dfrac{B - \sqrt{B^2 - 4AC}}{2} & B < 0 \end{cases}$$

$$r_s = \max\left\{\frac{Q}{A}, \frac{C}{Q}\right\}$$

$$CI = \max\left\{CS - 1.65PSN \bullet p_{sfc} \bullet r_s, 0\right\}$$

Then go back to repeat the iteration until a predefined iteration number is reached.

Finally, convert the unit of leaf stomatal resistance $r_s = r_s \bullet CF$

4.2.3 Javis stomatal resistance scheme

1) contribution due to incoming solar radiation

$$r_{c,s} = \frac{\left(\frac{2PAR}{RGL} + \frac{r_{s,\min}}{r_{s,\max}}\right)}{1 + \frac{2PAR}{RGL}}$$

$$r_{c,s} = \max\left\{r_{c,s}, 0.0001\right\}$$

2) contribution due to air temperature

$$r_{c,T} = 1 - 0.0016\left(T_{opt} - T_{sfc}\right)^2$$

$$r_{c,T} = \max\left\{r_{c,T}, 0.0001\right\}$$

3) contribution due to vapor pressure deficit

$$r_{c,q} = \frac{1}{1 + HS + \max\left\{q_{2,sat} - q_2, 0\right\}}$$

$$r_{c,q} = \max\left\{r_{c,q}, 0.01\right\}$$

Canopy resistance is determined due to all these factors

References

Bonan, G.B. (2008), *Ecological Climatology*: *Concepts and Applications* (2nd edition), Cambridge University Press, Cambridge, 550 pp.

Chen, F., Z. Janjic, and K. Mitchell (1997), Impact of atmospheric surface layer parameterization in the new land-surface scheme of the NCEP Mesoscale Eta numerical model, Bound.-Layer Meteor., 185, 391-421.

Łobocki, L. (1993), A Procedure for the Derivation of Surface-Layer Bulk Relationships from Simplified Second Order Closure Models, J. Appl. Meteorol., 32, 126–138.

Mellor, G. L., and T. Yamada (1982), Development of a turbulence closure model for geophysical fluid problems, Rev. Geophys., 20(4), 851–875, doi:10.1029/RG020i004p00851.

Paulson, C. A. (1970), The Mathematical Representation of Wind Speed and Temperature Profiles in the Unstable Atmospheric Surface Layer, J. Appl. Meteorol., 9, 857–861.

Mellor, G. L., and T. Yamada (1982), Development of a turbulence closure model for geophysical fluid problems, *Rev. Geophys*., 20(4), 851–875, doi:10.1029/RG020i004p00851.

5. Soil and Snow Temperature

5.1 Phase Change

5.1.1 Supercooled Water

There are two options available to calculate supercooled water: no iteration from *Niu and Yang* [2006], and Koren's iteration [*Koren et al.*, 2006] with Flerchinger's explicit solution ([*Flerchinger and Saxton*, 1989], introduced in [*Koren et al*., 2006]).

5.1.1.1 No Iteration

$$\Psi(T_i) = \frac{10^3 L_f (T_i - T_f)}{g T_i}, \qquad \text{if } T_i < T_f \tag{5-1}$$

Assume $\Psi(T_i) = \Psi(\theta_{liq})$

$$\theta_{liq,sat,i} = \theta_{sat,i} \left\{ \frac{10^3 L_f (T_i - T_f)}{g T_i \Psi_{sat,i}} \right\} \tag{5-2}$$

Where is $\theta_{liq,sat,i}$ supercooled liquid water content.

$$w_{liq,\max,i} = \theta_{sat,i} \Delta z_i \left\{ \frac{10^3 L_f (T_i - T_f)}{g T_i \Psi_{sat,i}} \right\}^{-1/B_i}, \qquad T_i < T_f \tag{5-3}$$

5.1.1.2 Iteration

1) Koren's Iteration

If $T_i < T_f$,

$$\theta_{liq,i} = \theta_i \tag{5-4}$$

$$\theta_{ice,i} = \theta_i - \theta_{liq,i} \tag{5-5}$$

Where $\theta_{ice,i}$ is initial guess of frozen content.

$$k_i = \ln\left(\frac{g\Psi_{sat,i}}{L_f}\right) \cdot (1 + c_k \theta_{ice,i})^2 \cdot \left(\frac{\theta_{sat,i}}{\theta_i - \theta_{ice,i}}\right)^{B_i} - \ln\left(-\frac{T_i - T_f}{T_i}\right) \tag{5-6}$$

$$D_{nom} = 2 \cdot \frac{c_k}{1 + c_k \theta_{ice,i}} + \frac{B_i}{\theta_i - \theta_{ice,i}} \tag{5-7}$$

$$\theta'_{ice,i} = \begin{cases} \theta_i - 0.02 & \text{if } \theta_{ice,i} - \dfrac{k_i}{D_{nom}} > \theta_i - 0.02 \\ 0 & \text{if } \theta_{ice,i} - \dfrac{k_i}{D_{nom}} > 0 \\ \theta_{ice,i} - \dfrac{k_i}{D_{nom}} & \text{otherwise} \end{cases} \quad (5\text{-}8)$$

Where $\theta'_{ice,i}$ is frozen content during each iteration.

$$\theta_{liq,sat,i} = \theta_i - \theta_{ice,i}$$

If more than 10 iterations, use explicit method (ck = 0 approximation). When $\theta'_{ice,i} - \theta_{ice,i} \le 0.005$, no more iterations required.

2) Explicit Solution

Equation 17 in *Koren et al*. [2006].

$$\theta_{liq,sat,i} = \theta_{sat,i}\left[\frac{L_f}{g\left(-\Psi_{sat,i}\right)}\frac{T_i - T_f}{T_i}\right]^{-1/B_i} \quad (5\text{-}9)$$

Where $0.02 \le \theta_{liq,sat,i} \le \theta_{sat,i}$.

$$w_{liq,\max,i} = 1000 \cdot \Delta z_i \theta_{liq,sat,i} \quad (5\text{-}10)$$

If there is ice in the coil and temperature is higher than freezing point, then ice is melting. If liquid water is more than supercooled water by mass, and soil temperature is lower than freezing point, then ice is accumulating. If snow exists, but its thickness is not enough to create a layer, snow is melting.

5.1.2 Energy Surplus and Loss for Melting and Freezing

$$\theta'_{ice,i} = \begin{cases} \dfrac{T_i - T_f}{f_i} & \text{no phase change} \\ 0 & \text{melting or freezing} \end{cases} \quad (5\text{-}11)$$

Where

$$f_i = \frac{\Delta t}{C \cdot \Delta z} \quad (5\text{-}12)$$

$$W_i = \frac{H_m \cdot \Delta t}{L_f} \quad (5\text{-}13)$$

5.1.3 The Rate of Melting and Freezing for Snow without a Layer

$$W_{sno}^{n+1} = W_{sno}^{n} - W_1 \geq 0 \quad (5\text{-}14)$$

$$H_r = H_i - \frac{L_f \left(W_{sno}^{n} - W_{sno}^{n+1}\right)}{\Delta t} \quad (5\text{-}15)$$

$$W_i = \begin{cases} \dfrac{H_r \cdot \Delta t}{L_f} & H_r > 0 \\ 0 & H_r \leq 0 \end{cases} \quad (5\text{-}16)$$

Snow melt

$$M_{1S} = \frac{\left(W_{sno}^{n} - W_{sno}^{n+1}\right)}{\Delta t} \geq 0 \quad (5\text{-}17)$$

$$E_{p,1S} = L_f M_{1S} \quad (5\text{-}18)$$

5.1.4 The Rate of Melting and Freezing for Snow and Soil

$$W_{ice,i}^{n+1} = W_{ice}^{n} - W_i \geq 0 \quad (5\text{-}19)$$

$$H_{r,i} = H_i - \frac{L_f \left(W_{ice,i}^{n} - W_{ice,i}^{n+1}\right)}{\Delta t} \quad (5\text{-}20)$$

$$W_{ice,i}^{n+1} = \min\left(W_{liq,i}^{n} + W_{ice,i}^{n}, W_{ice}^{n} - H_m\right) \quad (5\text{-}21)$$

$$w_{ice,i}^{n+1} = \begin{cases} \min\left(w_{liq,i}^{n} + w_{ice,i}^{n} - w_{liq,\max,i}^{n}, w_{ice}^{n} - H_m\right) & w_{liq,i}^{n} + w_{ice,i}^{n} \geq w_{liq,\max,i}^{n} \\ 0 & w_{liq,i}^{n} + w_{ice,i}^{n} < w_{liq,\max,i}^{n} \end{cases} \quad (5\text{-}22)$$

$$w_{liq,i}^{n+1} = w_{liq,i}^{n} + w_{ice,i}^{n} - w_{ice,i}^{n+1} \geq 0 \quad (5\text{-}23)$$

$$H_{r,i} = H_i - \frac{L_f \left(W_{ice,i}^{n} - W_{ice,i}^{n+1}\right)}{\Delta t} \quad (5\text{-}24)$$

If $|H_r| > 0$

$$T_i^{n+1} = \begin{cases} T_i^{n} + \dfrac{\theta}{\theta_{sat}} H_r & \text{soil layer} \\ T_f & \text{snow layer} \end{cases} \quad (5\text{-}25)$$

$$E_p = E_{p,1S} + \sum_{i=snl+1}^{N_{lvlgrrnd}} E_{p,i} \quad (5\text{-}26)$$

$$E_{p,i} = L_f \frac{\left(W_{ice,i}^{n} - W_{ice,i}^{n+1}\right)}{\Delta t} \tag{5-27}$$

For snow layers,

$$m_{ice,i} = w_{ice,i} \tag{5-28}$$

$$m_{liq,i} = w_{liq,i} \tag{5-29}$$

For soil layers,

$$\theta_{liq,i} = \frac{w_{ice,i}}{1000\Delta z} \tag{5-30}$$

$$\theta_i = \frac{w_{ice,i} + w_{liq,i}}{1000\Delta z} \tag{5-31}$$

5.2 Thermal Properties

5.2.1 Snow

Partial volume of ice in snow layer (the fraction of ice volume to snow volume)

$$\theta_{ice,i} = \frac{m_{ice,i}}{\Delta z_i \cdot \rho_{ice}} \tag{5-32}$$

Effective porosity

$$\theta_{e,i} = 1 - \theta_{ice,i} \tag{5-33}$$

Partial volume of liquid water in snow layer

$$\theta_{liq,i} = \frac{m_{liq,i}}{\Delta z_i \cdot \rho_{liq}} \tag{5-34}$$

Bulk density of snow

$$\rho_{snow,i} = \frac{m_{ice,i} + m_{liq,i}}{\Delta z_i} \tag{5-35}$$

Instead of using a constant of 0.525×10^6, volumetric specific heat is calculated by

$$C_{v,i} = C_{ice}\theta_{ice,i} + C_{liq}\theta_{liq,i} \tag{5-36}$$

There are several equations available for thermal conductivity of snow

$$k_i = 3.2217 \times 10^{-6} \rho_{snow,i}^2 , \qquad \text{[\textit{Yen}, 1965; \textit{Lynch-Stieglitz}, 1994]} \tag{5-37}$$

$$k_i = 2 \times 10^{-2} + 2.5 \times 10^{-6} \rho_{snow,i}^2 , \qquad \text{[\textit{Anderson}, 1976]} \tag{5-38}$$

$$k_i = 0.35 , \tag{5-39}$$

$$k_i = 2.576 \times 10^{-6} \rho_{snow,i}^2 + 0.074 , \qquad \text{[\textit{Verseghy}, 1991]} \tag{5-40}$$

$$k_i = 2.22 \left(\frac{\rho_{snow,i}}{1000} \right)^{1.88} , \qquad \text{[\textit{Yen}, 1981; \textit{Douville et al.}, 1995]} \tag{5-41}$$

Noah-MP uses the *Yen* [1965] method. However, CLM uses the equation from *Jordan* [1991] (see CLM 4.0 Technical Note). NCAR Land Surface Model 1.0 uses the combination of *Lunardini* [1981] and *Farouki* [1981] (see NCAR LSM 1.0 Technical Note).

5.2.2 Soil

Soil ice content

$$\theta_{ice,i} = \theta_i - \theta_{liq,i} \tag{5-42}$$

Heat capacity

$$C_i = \theta_{liq,i} C_{liq} + (1 - \theta_{sat}) C_{soil} + (\theta_{sat} - \theta_i) C_{air} + \theta_{ice,i} C_{ice} \tag{5-43}$$

Saturation ratio

$$S_{r,i} = \frac{\theta_i}{\theta_{sat}} \tag{5-44}$$

Thermal conductivity for the solids

$$k_{sld} = k_{qtz}^{\ f_{qtz}} \cdot k_o^{\ 1-f_{qtz}} \tag{5-45}$$

Where k_{sld} is thermal conductivity for the solids (W m^{-1} K^{-1}). k_{qtz} is thermal conductivity for quartz (W m^{-1} K^{-1}), k_{qtz} = 7.7 W m^{-1} K^{-1}. K_o is thermal conductivity for the other soil component (W m^{-1} K^{-1}), k_o = 2.0 W m^{-1} K^{-1}. f_{qtz} is the soil quartz content which depends on soil type and is stored in soil parameter table.

Unfrozen fraction, from 1, i.e., 100% liquid, to 0 (100% frozen).

$$f_{uf_i} = \frac{\theta_{liq,i}}{\theta_i} \tag{5-46}$$

Unfrozen volume for saturation

$$\theta_{uf,sat,i} = f_{uf,i} \theta_{sat} \tag{5-47}$$

Saturated thermal conductivity

$$k_{sat,i} = k_{sld}^{\ (1-\theta_{sat})} \cdot k_{ice}^{\ (\theta_{sat}-\theta_{uf,sat,i})} \cdot k_{liq}^{\ \theta_{uf,sat,i}} \tag{5-48}$$

Where k_{ice} is thermal conductivity of ice (W m^{-1} K^{-1}), k_{ice} = 2.2 W m^{-1} K^{-1}. K_{liq} is water thermal conductivity (W m^{-1} K^{-1}), k_{ice} = 0.57 W m^{-1} K^{-1}.

Dry density in unit of kg m^{-3},

$$\rho_{dry} = 2700(1 - \theta_{sat}) \tag{5-49}$$

Dry thermal conductivity W m^{-1} K^{-1},

$$k_{dry} = \frac{0.135\rho_{dry} + 64}{2700 - 0.947\rho_{dry}} \tag{5-50}$$

Kersten number k_e, as a function of the saturation S_r and phase of water, uses "fine" formula, valid for soils containing at least 5% of particles with diameter less than 2.0×10^{-6} m [*Peters-Lidard et al.*, 1998]. It is calculated by

$$k_{e,i} = \begin{cases} S_{r,i}, & \text{For frozen soil } (\theta_{\text{liq}} + 0.0005 < \theta) \\ \ln S_{r,i} + 1.0, & \text{For unfrozen soil and } S_r > 0.1 \\ 0 & \text{For unfrozen soil and } S_r \le 0.1 \end{cases} \tag{5-51}$$

Thermal conductivity of soil,

$$k_i = k_{e,i}\left(k_{sat} - k_{dry}\right) + k_{dry} \tag{5-52}$$

5.2.3 Lake

Heat capacity

$$C = \begin{cases} C_{liq}, & \text{liquid} \\ C_{ice}, & \text{ice} \end{cases} \tag{5-53}$$

Thermal conductivity

$$k_i = \begin{cases} k_{liq}, & \text{liquid} \\ k_{ice}, & \text{ice} \end{cases} \tag{5-54}$$

Calculating a temporary factor used in phase change

$$f_{temp,i} = \frac{\Delta t}{C \cdot \Delta z_i} \tag{5-55}$$

snow/soil interface

$$k_1 = \frac{k_{soil}\Delta z_1 + k_{snow} z_{snow}}{\Delta z_i + z_{snow}} \tag{5-56}$$

Where k_{soil} is the thermal conductivity calculated for layer 1 (top soil layer). Snow thermal conductivity k_{snow} is constrained to 0.35 W m^{-1} K^{-1}. z_{snow} is snow height.

5.3 SNOW WATER (this module is used to calculate the snow water equivalent)

Computer snow (up to 3L) and soil (4L) temperature, snow water is predicted from a multi-layer model. Snow accumulation/ablation parameterizations of the Noah-MP model are based on mass and energy balance in the snowpack. The change in snowpack snow water equivalent is balanced by the input snowfall, and output snowmelt and snow sublimation:. Snow accumulation/ablation parameterization of the original Eta model [*Chen et al*., 1996] is based on the energy and mass balance of the snowpack and the snowmelt rate (M_s) *is determined by*

$$\frac{dW_s}{dt} = P_s - M_s - E \qquad (1)$$

$$M_s = \frac{1}{L}\left(Q_{sw} + Q_{lw} - Q_{lt} - Q_{sn} - Q_g\right) \qquad (2)$$

Where W_s is the snow water equivalent, P_s is precipitation in the form of snow, M_s is the snowmelt rate, E is the snow evaporation, Q_{sw} is net solar radiation, Q_{lw} is net longwave radiation, Q_{lt} is the latent heat flux,Q_{sn} is the sensible heat flux,Q_g is ground heat flux, and L is the latent heat of fusion.

The parameterization neglects heat transferred by movement of meltwater in the snowpack and assumes that all liquid water covered area(except for the snow albedo). Snowpack physical characteristics, thermal conductivity K_s, and density $\boldsymbol{\rho_s}$are assumed constant at 0.35 $Wm^{-1}K^{-1}$ AND 0.4 GCM^{-3}, RESPECTIVELY. Below, it will be shown that this assumption can lead to significant overestimation of snow depth.

From code： snow water (5 subroutines)

Subroutne1: for the snowfall (it happens when there is new snowfall)

Subroutine 2: compact (Calculate the pressure of overlying snow [kg/m2])

Subroutine 3 : combine (snow melting and sublimation)

Subroutine 4: divide (?? subdivide a specify snow layer when the snow depth is over limitation)

Subroutine 5: snowh20 (to obtain equilibrium state of snow in glacier region)

$Sneqv_{i+1}$=$sneqv_i$+snice+snliq ,i=isnow+1,0 (1.1) (main equation)

Subroutne1: for the snowfall (it happens when there is new snowfall)

It is used to calculate the snow depth and density which induced by the new snowfall. The value of snow depth and density returned.

shallow snow/no layer(newcode=0)

Snowh=$snowh_o$+snowhin*dt (1.1.1)

Sneqv =$sneqv_o$+qsnow*dat (1.1.2)

When snowh >=0.05 then set newcode=1(creating a new layer)

Isnow=-1 (it has one layer of new snowfall)

Dzsnso (0)=showh

Snice(0)=sneqv

Then when the depth of snowfall over one layer, the snice and dzsns will be calculated (snow with layers)

$Snice(isnow+1)=since_o(isnow+1)+qsnow*dt$ (1.1.3)

$Dzsnso(isnow+1)=dzsnso_o(isnow+1)+snowhin*dt$ (1.1.4)

When isnow <0 the snow water will used subroutine 2-4 (when snow depth is more than one layer)

Subroutine 2: compact (it is used to calculate dzsnso)

Calculate the pressure of overlying snow [kg/m2]

Wx=snice +snliq (1.2.1)

Fice=snice/wx (1.2.2)

Void=1.-(snice/denice+snliq/denh20)/dzsnso (1-fraction of quality of ice and liquid in snow layer) (1.2.3)

(allow compaction only for non-saturated node and higher ice lens node)

We are first setting the initial value of ddz1 as -2.5e-6*exp(-0.04*(273.15-stc))When void >0.01 and snice>0.1 (when fraction of snow layer ice to snow thickness)

$$\text{Ddz1}=\begin{cases} \text{ddz1} * \exp\left(-46.0e-3 * \left(\frac{since}{dzsnso} - dm\right)\right) & , \frac{since}{dzsnso} - dm > 0 \\ \text{ddz1} * 2.0 & , \text{snliq} > 0 * dzsnso \text{ , } liquid\ water\ term \end{cases} \quad (1.2.4)$$

Ddz2=-(burden+0.5*wx)*exp(-0.08*(stc-tfrz)-(2.1e-3)*since/dzsnso)/（0.8e+6) (1.2.5)

$$\text{Ddz3}=-\frac{\max\left(0.,\frac{\text{ficeold}-\text{fice}}{\max(1.E-6,\text{ficeold})}\right)}{\text{dt}} \quad \text{compacting occuring during melt} \quad (1.2.6)$$

Time rate of fractional change in DZ

pdzdtc=max(-0.5,(ddz1+ddz2+ddz3)/dt) (1.2.7)

$Dzsnso=dzsnsd_o*(1.+pdzdtc)$ (1.2.8)

Subroutine 3: combine (when it happens snow melting and surface sublimation)

$Sh20=sh20_o+snliq/(dzsnso*1000)$

Sice=sice+snice/(dzsnso*1000)

When there is too large surface sublimation, we need to conserve water first.

Sh20=sh20+sice

When all snow gone, the liquid water was assumed to be ponded on soil surface

Sh20=sh20+zwliq/(dzsnso(1)*1000.)

We will use subroutine combo when combined node I and j and then storied as node j

Then shift all elements in the above layer to down one

Subroutine 4: divide (??)

Subroutine 5: snowh20 (renew the mass of ice lens (snice) and liquid (snliq) of the surface snow layer resulting from sublimation (frost)/evaporation(dew))

Bdsnow=snice/dzsnso

Snoflow=sneqv-2000.

Snice=snice-snoflow

It has been divided into 2 different case,

Case I ,for shallow snow without layer, snow surface sublimation may be larger than existing snow mass. To conserve water, excessive sublimation is used to reduce soil water. Smaller time steps would tend to avoid this problem

$$\left\{\begin{array}{c} \mathrm{temp} = \mathrm{sneqv} \\ \mathrm{sneqv} = \mathrm{sneqv} - \mathrm{qsnsub} * \mathrm{dat} + \mathrm{qsnfor} * \mathrm{dt} \\ \mathrm{propor} = \frac{\mathrm{sneqv}}{\mathrm{temp}} \\ \mathrm{snowh} = \max(0., \mathrm{propor} * \mathrm{snowh}) \end{array}\right\} \quad \text{isnow=0 and dneqv>0}$$

$$\left\{\begin{array}{c} \mathrm{sice}(1) = \mathrm{sice}(1) + \frac{\mathrm{sneqv}}{\mathrm{dzsnso}(1) * 1000} \\ \mathrm{sneqv} = 0 \end{array}\right\} \quad \text{sneqv<0}$$

$$\left\{\begin{array}{c} \mathrm{sh20}(1) = \mathrm{sh20}(1) + \mathrm{sice}(1) \\ \mathrm{sice}(1) = 0. \end{array}\right\} \text{sice<0,}$$

Case Ⅱ,For deep snow

Wgdif=snice(isnow+1)-qsnsub*dt+qsnfro*dt

Snice(isnow+1)=wgdif

It calls subroutine combine when wgdif <1.e-6 and isnow<0

Snliq (isnow+1)=snliq(isnow+1)+qrain*dt

Snliq(isnow+1)=max(0.,snliq(isnow+1))

Calculate porosity and partial volume

$$\mathrm{vol\_ice(j)} = \min\left(1.,\frac{\mathrm{snice(j)}}{\mathrm{dzsnso(j)8denice}}\right)$$

Epore(j)=1.-vol_ice(j)   j>=isnow+1

$Sice=sice_o+sneq/(dzsnso*1000)$

……….

The liquid water from snow bottom to soil

Qsnbot=qout/dt

5.4 Snow and Frozen Soil Temperature

Snow-skin temperatures in the vegetated fraction ($T_{g,v}$) and bare fraction ($T_{g,b}$) are solved iteratively through the energy balance equations (1) and (2),

$$F_{veg} S_{ag} = F_{veg} (L_{ag,v} (T_{g,v}) + LE_{g,v} (T_{g,v}) + H_{g,v} (T_{g,v}) + G_v (T_{g,v})) \quad (2.1)$$

The ground-absorbed solar radiation over the gridcell, S_{ag}, is shared by the vegetated ground with an amount of $S_{ag}F_{veg}$ and the bare ground with an amount of $S_{ag}(1–F_{veg})$. The vegetated ground emits longwave radiation to the canopy and exchanges latent heat ($LE_{g,v}$) and sensible heat ($H_{g,v}$) fluxes with the canopy air and ground heat with the upper soil (G_v) at a temperature $T_{g,v}$.

$$(1 - F_{veg}) S_{ag} = (1 - F_{veg})(L_{ag,b} (T_{g,b}) + LE_{g,b} (T_{g,b}) + H_{g,b} (T_{g,b}) + G_b (T_{g,b})) \quad (2.2)$$

where $L_{ag,v}$ is the net longwave radiation (positive upward) absorbed by the vegetated ground. Analogously, the bare ground at the fractional area, $1–F_{veg}$, emits longwave radiation to the atmosphere and exchanges latent heat ($LE_{g,b}$) and sensible heat ($H_{g,b}$) with the atmosphere at a temperature $T_{g,b}$.

The G in the equator 3 is regarded as the upper boundary condition of the snow/soil temperature equation, the temperatures of the snow and soil layers are then solved together through one tri-diagonal matrix with its dimension varying with the total number of snow and soil layers.

5.4.1 Numerical Solution

The soil column is discretized into 4 layers and snowpack can be divided by up to three layers depending on the total snow depth h_{sno} ,as in Yang and Niu[2003]. The layers from top to bottom are indexed in the fortran code as i=-2,-1,0.Layer i=0 is the snow layer next to the top of the soil surface and layer i=isnow+1 is the top layer, where the variable snl is the negative of the number of snow layers.The number of snow layers and the thickness of each layer is a function of snow depth h_{sno} as follows:

Where $h_{sno} < 0.045m$ is the total snow depth. There is no snow layer exists and the snowpack is combined with the top soil layer.

Where $0.05 \ge h_{sno} \ge$ *0.045 m, the first snow layer* $\Delta z_i(m)$ *is*

$$\Delta z_i = \{h_{sno}\} \qquad (2.3)$$

Where $0.1 \ge h_{sno} \ge$ *0.05 m, two snow layers are created and the thickness of each layer* Δz_i *is*

$$\Delta z_i = \begin{Bmatrix} h_{sno}/2, i = -1 \\ h_{sno}/2, i = 0 \end{Bmatrix} \qquad (2.4)$$

Where $0.15 \ge h_{sno} \ge 0.1$ m, the two-layer thicknesses are:

$$\Delta z_i = \begin{Bmatrix} 0.05, i = -1 \\ (h_{sno} - \Delta z_{-1}), i = 0 \end{Bmatrix} \qquad (2.5)$$

Where $0.45 \ge h_{sno} \ge 0.15$ m, a third layer is created; the three layer thicknesses are:

$$\Delta z_i = \begin{Bmatrix} 0.05, i = -2 \\ \left(h_{sno} - \Delta z_0\right)/ 2, i = -1 \\ \left(\left(h_{sno} - \Delta z_0\right)/ 2, i = 0\right) \end{Bmatrix} \quad (2.6)$$

Where $h_{sno} \geq 0.45$ m, the layer thicknesses for the three snow layers are: $\Delta z_{-2} = 0.05$ m, $\Delta z_{-1} = 0.2$ m, and $\Delta z_0 = (h_{sno} - \Delta z_{-2} - \Delta z_{-1})$ m.

$$\Delta z_i = \begin{Bmatrix} 0.05, i = -2 \\ 0.2, i = -1 \\ \left(\left(h_{sno} - \Delta z_0 - \Delta z_{-1}\right), i = 0\right) \end{Bmatrix} \quad (2.7)$$

If a layer thickness is less than its minimum value (0.045 m, 0.05 m, and 0.2 m for the three layers from top to bottom) due to sublimation and/or melt, the layer is combined with its lower neighboring layer; the layers are then re-divided depending on the total snow depth following the above procedure. The thinner first snow layer is designed to more accurately resolve the ground heat flux.

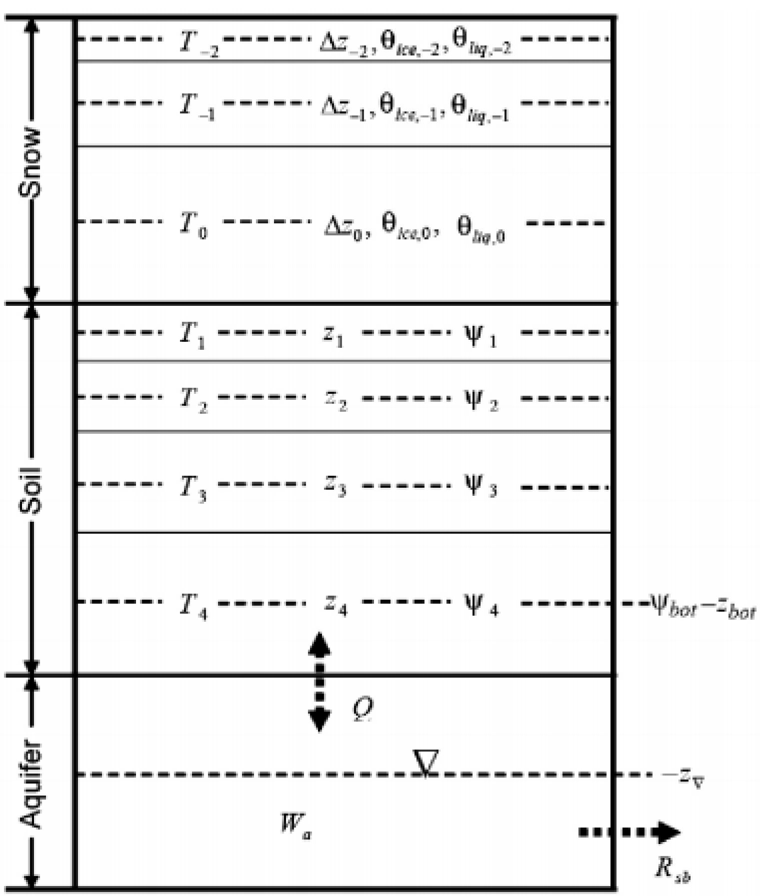


Figure 2. Schematic diagram for snow, soil, and an unconfined aquifer as represented in the model. The indices for the snow layers from the top are −2, −1, and 0 to continuously transition to soil layer's indices 1, 2, 3, and 4. The variables are described in detail in the text.

5.4.2 Phase Change (melting occurs)

Snow and soil layer temperatures are then used to assess the energy for melting or freezing ($H_{m,i}$) for the *i*th snow and soil layers, i.e., the energy excess or deficit needed to change a snow or soil layer temperature to the freezing point T_{frz}:

$$H_{m,i} = C_i \Delta z_i \frac{T_i^{N+1} - T_{frz}}{\Delta t} \qquad i = isno + 1, 4 \tag{2.8}$$

Where energy for melting or freezing ($H_{m,i}$) , T_i^{N+1} is the *i*th layer snow or soil temperature solved through the tri-diagonal matrix (T_i^{N+1} can be greater than T_{frz} during midday hours in the melting season before the treatment of phase change). Δz_i and Δt are layer thickness and time step. Subscript "*isno*" represents the total number of snow layers in a negative number (for instance, when there are three snow layers, *isno* = –3; *isno*+1= –2 represents the surface snow layer). C_i is the volumetric heat capacity:

$$C_i = \begin{cases} C_{ice}\theta_{ice,i} + C_{liq}\theta_{liq,i} & i = isno + 1, 0 \\ C_{ice}\theta_{ice,i} + C_{liq}\theta_{liq,i} + C_{soil}(1-\theta_{sat}) & i = 1, 4 \end{cases} \tag{2.9}$$

where $\theta_{ice,i}$ and $\theta_{liq,i}$ stand for partial volume of ice and liquid water in the *i*th snow or soil layer (Figure 2), and C_{ice} and C_{liq} for volumetric heat capacity for ice and liquid water, respectively. θ_{sat} is soil porosity, and C_{soil} is the volumetric heat capacity of soil particles.

When a snow or soil layer's ice content $\theta_{ice,i} > 0$ and $T_i^{N+1} > T_{frz}$, melting occurs. In the melting phase,

$\theta_{ice,i} > 0$ and $T_i^{N+1} > T_{frz}, melting, i = isnow + 1{,}0$ H_m (>0)

$T_i^{N+1} < T_{frz}$ and $\theta_{liq,i} > 0$ (for snow) or $\theta_{liq,i} > \theta_{liq\max,i}$ (for soil),freezing,i=0,isoil

$$H_m = L_f \theta_{ice,i} \rho_{ice} \Delta z_i / \Delta t , \quad (2.10)$$

When H_m >0,where L_f and ρ_{ice} are latent heat of fusion (= 0.3336 × 10^6 J kg^{-1}) and ice density (= 917 kg m^{-3}). The $\theta_{liq,i}$ is limited by its maximum value of a snow layer (or holding capacity, $\theta_{liq\,\max,i}$ = 0.03 m^3 / m^3); excessive $\theta_{liq,i}$ above $\theta_{liq\,\max,i}$ flows down to its lower neighboring layer and eventually to the soil surface. When freezing occurs, where $\theta_{liq,\max,i}$ is the upper limit of the supercooled liquid water (see section 4.6 for details).

When H_m (< 0) is limited by the latent heat released by freezing all the liquid water in a snow layer or the liquid water over $\theta_{liq,\max,i}$ in a soil layer within one time step. The residual energy that may not be consumed by melting or released from freezing is used to heat or cool the snow or soil layer.

5.4.3 Snow Interception Model

We further implemented a snow interception model [*Niu and Yang*, 2004] into the Noah model. Because the interception capacity for snowfall is much greater than that for rainfall, interception of snowfall by the canopy and subsequent sublimation from the canopy snow may greatly reduce the snow mass on the ground.

The snow cover fraction (SCF) on the ground, $f_{sno,g}$, is parameterized as a function of snow depth, ground roughness length, and snow density following *Niu and Yang* [2007].

$$\alpha_g = (1 - f_{sno,g})\alpha_{soi} + f_{sno,g}\alpha_{sno}\,. \qquad (2.11)$$

The ground surface albedo α_g, is then parameterized as an area-weighted average of albedos of snow (α_{sno}) and bare soil (α_{soi}). The SCF of the canopy ($f_{sno,c}$) adopts the formulation of *Deardorff* [1978] for the wetted fraction of the canopy, depending on snow mass on the canopy. It is used as a weight to average the scattering parameters used in the two-stream approximation over fractional snow-covered canopy ($f_{sno,c}$) and non-covered canopy ($1-f_{sno,c}$) .

References

Chen, F., K. Mitchell, J. Schaake, Y. Xue, H. Pan, V, Koren, Y. Duan, M. Ek, and A, Betts (1996), Modeling of land-surface evaporation by four schemes and comparison with FIFE observations, *J. Geophys. Res.*,101, 7251-7268.

Douville, H., J.-F. Royer, and J.-F. Mahfouf (1995) A new snow parameterization for the Météo-France climate model. Part I: validation in stand-alone experiments, Climate Dynamics, 12, 21-35.

Flerchinger, G.N. and K.E. Saxton (1989), Simultaneous heat and water model of a freezing snow-residue-soil system I. Theory and development, *Trans. ASAE*, 32(2), 565-571.

Koren, V., J. Schaake, K. Mitchell, Q.-Y. Duan, F. Chen, and J. M. Baker (1999), A parameterization of snowpack and frozen ground intended for NCEP weather and climate models, *J. Geophys. Res*., 104(D16), 19569–19586.

Niu, G.-Y. and Z.-L. Yang, 2006: Effects of frozen soil on snowmelt runoff and soil water storage at a continental scale, *Journal of Hydrometeorology*, 7 (5), 937-952.

Niu, G.-Y., et al. (2011), The community Noah land surface model with multiparameterization options (Noah‐MP): 1. Model description and evaluation with local‐scale measurements, *J. Geophys. Res*., 116, D12109.

Lynch-Stieglitz, M. (1994), The development and validation of a simple snow model for the GISS GCM, J. Climate, 7, 1842-1855.

Yen, Y. C. (1965), Heat transfer Characteristics of Naturally Compacted Snow. U.S. Army Cold Regions Research and Engineering Laboratory. Research Report 166, Hanover, NH, 9 pp.

Yen, Y.-C., (1981), Review of thermal properties of snow, ice and sea ice, Cold Regions Research and Engineering Laboratory (CRREL) Report 81-10, U.S. Army Cold Reg. Res. and Eng. Lab., Hanover, NH.

6. Hydrology

6.1 Summary

The water cycle processes considered in the model include surface runoff, subsurface runoff, infiltration, groundwater, snow accumulation and melt, interception, throughfall, and the redistribution within the soil column to simulate canopy water W_{can}, snow water equivalent W_{sno}, and soil water $\sum_i \theta_i \Delta z_i$, where θ_i is the volumetric soil water content and Δz is the soil thickness (m). All water fluxes are in units of mm s^{-1}.

If snow exists, snow surface sublimation rate (mm s^{-1}), q_{subl}

$$q_{subl} = \min\left(q_{vap}, \frac{W_{sno}}{\Delta t}\right) \tag{6-1}$$

Where q_{vap} is soil surface evaporation rate (mm s^{-1}).

$$q_{seva} = q_{vap} - q_{subl} \tag{6-2}$$

Where q_{seva} is soil surface evaporation rate adjusted for sublimation from the snow pack (mm s^{-1}).

$$q_{fro} = q_{dew} \tag{6-3}$$

Where q_{fro} is snow surface frost rate (mm s^{-1}). q_{dew} is soil surface dew rate (mm s^{-1}).

$$q_{sdew} = q_{dew} - q_{fro} \tag{6-4}$$

Where q_{sdew} is soil surface dew rate adjusted for frost (mm s^{-1}). When snow exists, dew water is added as frost to the snow pack and q_{seva} becomes 0.

$$q_{insur} = \begin{cases} \frac{W_{pond}}{\Delta t} + q_{snbot} + q_{sdew} + q_{rain} & \text{if no snow layer} \\ \frac{W_{pond}}{\Delta t} + q_{snbot} + q_{sdew} & \text{if snow layer exists} \end{cases} \tag{6-5}$$

Where q_{insur} is water input on soil surface (mm s^{-1}). q_{snbot} is melting water out of snow bottom (mm s^{-1}). q_{rain} is rain at ground surface (mm s^{-1}).

If the surface type is lake,

$$q_{sur} = \begin{cases} q_{insur} & \text{if } W'_{lake} > W_{lake,\max} \\ 0 & \text{otherwise} \end{cases} \tag{6-6}$$

Where q_{sur} is surface runoff (mm s^{-1}). W'_{lake} is water storage in lake from previous time step (mm). $W_{lake,max}$ is maximum water storage in lake (mm), $W_{lake,max}$ =5000 mm.

$$W_{lake} = W_{lake}^{'} + \left(q_{infur} - q_{seva} - q_{sur}\right)\Delta t \quad (6\text{-}7)$$

Where W_{lake} is updated water storage in lake (mm).

If free drainage option is selected for runoff and groundwater,

$$q_{sub} = q_{sub}^{'} + q_{drain} \quad (6\text{-}8)$$

Where q_{sub} is updated subsurface runoff (mm s^{-1}). q'_{sub} is subsurface runoff from previous time step(mm s^{-1}). q_{drain} is soil-bottom free drainage (mm s^{-1})

6.2 Canopy Hydrology

6.2.1 Partitioning Precipitation into Rainfall and Snowfall

For partitioning precipitation into rain or snowfall there are three options that can be selected: 1) is based on Jordan (1991) [add information] 2) is based on BATS when the surface temperature is the freezing temperature added to 2.2 degrees, and 3) is the surface temperature times the freezing temperature. Option 1 is suggested for the majority of work. If option 1 is chosen, the following is carried out:

$$F_{p,ice} = \begin{cases} 0 & T_{sfc} > T_{frz} \\ 1 & T_{sfc} \leq T_{frz} + 0.5 \\ 1 - \left(-54.632 + 0.2 * T_{sfc}\right) & T_{sfc} \leq T_{frz} + 2 \\ 0.6 & T_{sfc} > T_{frz} + 0.5 \end{cases} \quad (6\text{-}9)$$

If option 2 is selected the following is expected:

$$F_{p,ice} = \begin{cases} 0 & T_{sfc} \geq T_{frz} + 2.2 \\ 1 & T_{sfc} < T_{frz} \end{cases} \quad (6\text{-}10)$$

If option 3 is desirable the following will occur:

$$F_{p,ice} = \begin{cases} 0 & T_{sfc} \geq T_{frz} \\ 1 & T_{sfc} < T_{frz} \end{cases} \quad (6\text{-}11)$$

6.2.2 Fresh Snow Density

The Hedstrom NR and JW Pomeroy (1998), Hydrologic Processes, 12, 1611-1625 was used for fresh snow density, ρ_s. It is based on the idea that Bulk density snowfall BD_{fall} is affected by air temperature T_{air} and rain R as well as snowfall S is dependent on convective P_{conv} and large scale P_{syn} precipitation. This precipitation is treated differently depending on how much of the precipitation fraction is ice $F_{p,ice}$.

$$\rho_s = 67.92 + 51.25\, e^{\frac{(T_{sfc} - T_{frz})}{2.59}} \quad (6\text{-}12)$$

$$BD_{fall} = \begin{cases} 120 & 120 > \rho_s \\ \rho_s & 120 < \rho_s \end{cases} \quad (6\text{-}13)$$

$$R = (P_{conv} + P_{syn}) * (1 - F_{p,ice}) \quad (6\text{-}14)$$

$$S = (P_{conv} + P_{syn}) * (F_{p,ice}) \quad (6\text{-}15)$$

6.2.3 Fractional area that receives precipitation

The fractional area that receives precipitation F_p is based on Niu et al. 2005 and is the result of convective precipitation P_{conv} and large scale precipitation P_{syn}.

$$F_p = \left\{ \frac{P_{conv}+P_{syn}}{10*P_{conv}+P_{syn}} \quad P_{conv} + P_{syn} > 0 \right\} \quad (6\text{-}16)$$

1.2 (if we do the 1.2.3) Liquid water:

6.2.4 Maximum Canopy Water

The maximum amount of water within the canopy P_{max} is determined by the maximum intercepted water P_{maxh2o}, the vegetation type VEG_{typ}, the Leaf Area Index , and the Stem Area Index SAI.

$$P_{max} = P_{maxh2o}(VEG_{typ}) * (LAI + SAI) \quad (6\text{-}17)$$

Average interception and through fall, if there is a canopy, ie LAI and SAI are greater than zero, expect the expressions below where interception of rain is R_{int}, green vegetation fraction is F_{veg}, rainfall is R, the fraction of a grid cell that receives precipitation is F_p, the maximum canopy water is M_{liq}, the canopy liquid intercepted water is C_{liq}, and the time step is Δt:

$$R_{int} = F_{veg} * R * F_p \quad (6\text{-}18)$$

$$R_{int} = \begin{cases} R_{int} & R_{int} < \frac{M_{liq}-C_{liq}}{\Delta t} * 1 - e^{-\frac{R*\Delta t}{M_{liq}}} \\ \frac{M_{liq}-C_{liq}}{\Delta t} * 1 - e^{-\frac{R*\Delta t}{M_{liq}}} & R_{int} > \frac{M_{liq}-C_{liq}}{\Delta t} * 1 - e^{-\frac{R*\Delta t}{M_{liq}}} \end{cases} \quad (1\text{-}11)$$

$$R_{int} = \begin{cases} R_{int} & R_{int} > 0 \\ 0 & R_{int} < 0 \end{cases} \quad (6\text{-}19)$$

$$R_{drip} = F_{veg} * R - R_{int} \quad (6\text{-}20)$$

$$R_{throu} = (1 - F_{veg}) * R \quad (6\text{-}21)$$

If there is no canopy, the through fall will be equal to the rain.

6.2.5 Evaporation, transpiration, and dew

If the temperature of the veg T_v is not freezing ($T_v > T_{frz}$) then the following is what is expected for evaporation R_{eva}, transpiration ET, and dew formation R_{dew}. Evaporation flux is represented by ET_{flux}, latent heat of varporization is represented by L_v, and canopy evaporation is denoted by C_{ev}.

$$ET = \begin{cases} \frac{ET\ flux}{L_v} & \frac{ET\ flux}{L_v} > 0 \\ 0 & \frac{ET\ flux}{L_v} < 0 \end{cases} \tag{6-22}$$

$$R_{eva} = \begin{cases} \frac{C_{ev}}{L_v} & \frac{C_{ev}}{L_v} > 0 \\ 0 & \frac{C_{ev}}{L_v} < 0 \end{cases} \tag{6-23}$$

$$R_{dew} = \left| \begin{cases} \frac{C_{ev}}{L_v} & \frac{C_{ev}}{L_v} < 0 \\ 0 & \frac{C_{ev}}{L_v} > 0 \end{cases} \right| \tag{6-24}$$

Because the vegetation temperature is above freezing, $T_v > T_{frz}$, no sublimation Q_{sub} or frost R_{frost} formation occurs. If the vegetation temperature is below freezing, $T_v < T_{frz}$, the following is expected as a result where latent heat of sublimation is L_s .

$$ET = \begin{cases} \frac{ET\ flux}{L_s} & \frac{ET\ flux}{L_s} > 0 \\ 0 & \frac{ET\ flux}{L_s} < 0 \end{cases} \tag{6-25}$$

$$Q_{sub} = \begin{cases} \frac{ET\ flux}{L_s} & \frac{ET\ flux}{L_s} > 0 \\ 0 & \frac{ET\ flux}{L_s} < 0 \end{cases} \tag{6-26}$$

$$R_{frost} = \left| \begin{cases} \frac{ET\ flux}{L_s} & \frac{ET\ flux}{L_s} < 0 \\ 0 & \frac{ET\ flux}{L_s} > 0 \end{cases} \right| \tag{6-27}$$

6.2.6 Canopy Water Balance

It is most convenient to allow dew to bring canopy liquid C_{liq} above the maximum water or else re-adjustments must be made to drip. Evaporation rate is R_{eva} .

$$R_{eva} = \begin{cases} \frac{C\ liq}{\Delta t} & \frac{C\ liq}{\Delta t} < R_{eva} \\ R_{eva} & \frac{C\ liq}{\Delta t} > R_{eva} \end{cases} \tag{6-28}$$

The canopy liquid intercepted water C_{liq} is determined by the rain intercepted R_{int} , the dew formation R_{dew}, and the amount of evaporation E occurring over time t .

$$C_{liq} = \begin{cases} 0 & 0 > C_{liq} + (R_{int} + R_{dew} - E)\Delta t \\ C_{liq} + (R_{int} + R_{dew} - R_{eva})\Delta t & 0 < C_{liq} + (R_{int} + R_{dew} - E)\Delta t \end{cases} \tag{6-29}$$

$$C_{liq} = \begin{cases} 0 & C\ _{liq} \geq 1 \quad * 10^{-3} \end{cases} \tag{6-30}$$

The intercepted water by the canopy must be above a threshold of 1E-3 in order for the model to consider canopy liquid present.

The Maximum canopy capacity for snow interceptionM_{snow}, is dependent on LAI, SAI, and the bulk density snowfall BD_{fall}, it is treated as $M_{snow} = 6.6(\frac{0.27+46}{BD_{fall}})(LAI + SAI)$ within Noah-MP's Canopy Water subroutine.

If there is a canopy, i.e. LAI and SAI > 0, then the following is also true where R_{sint} is intercepted snow, S is snowfall, F_{veg} is green vegetation fraction, F_p is the fraction of the grid cell which is receiving precipitation, and C_{aice} is canopy intercepted ice mass:

$$R_{sint} = F_{veg} * S * F_p \tag{6-31}$$

$$R_{sint} = \begin{cases} R_{sint} & R_{sint} < (M_{snow} + C_{aice})\Delta t * \left(1 - e^{-\frac{S\Delta t}{M_{snow}}}\right) \\ (M_{snow} + C_{aice})\Delta t * \left(1 - e^{-\frac{S\Delta t}{M_{snow}}}\right) & R_{sint} > (M_{snow} + C_{aice})\Delta t * \left(1 - e^{-\frac{S\Delta t}{M_{snow}}}\right) \end{cases} \tag{6-32}$$

$$R_{sint} = \begin{cases} R_{sint} & R_{sint} > 0 \\ 0 & R_{sint} < 0 \end{cases} \tag{6-33}$$

The temperature factor for unloading rate T_{fu} is treated as the maximum between vegetation temperature T_v and …(add)

$$T_{fu} = \begin{cases} 0 & 0 > \frac{(T_v - 270.15)}{1.87*10^5} \\ \frac{(T_v - 270.15)}{1.87*10^5} & 0 < \frac{(T_v - 270.15)}{1.87*10^5} \end{cases} \tag{6-34}$$

The frictional velocity $V_f$is dependent on the U and V component of the wind: (check syntax)

$$V_f = \sqrt{\frac{U^2 + V^2}{1.56*10^5}} \tag{6-35}$$

The drip rate of snow $R_{sdrip}$is a function of canopy ice C_{aice}, the time step Δt, the frictional velocity V_f, and the temperature factor for unloading rate T_{fu}.

$$R_{sdrip} = \begin{cases} 0 & 0 > \left(\frac{C_{aice}}{\Delta t}\right) * \left(V_f + T_{fu}\right) \\ \left(\frac{C_{aice}}{\Delta t}\right) * \left(V_f + T_{fu}\right) & 0 < \left(\frac{C_{aice}}{\Delta t}\right) * \left(V_f + T_{fu}\right) \end{cases} \tag{6-36}$$

The canopy through fall of snow R_{sthrou} is dependent on the green vegetation fraction F_{veg}, the snowfall P_s, and the intercepted snow from the canopy R_{sint}.

$$R_{sthrou} = (1 - F_{veg})P_s + (F_{veg} * P_s - R_{sint}) \tag{6-37}$$

If there is no canopy, the trough fall is just equal to the snowfall. The sublimation rate Q_{sub} in this case is a function of canopy ice C_{aice} and time and the canopy ice depends on the intercepted canopy snow R_{sint}, the drip rate R_{sdrip}, the frost formation Q_{frost}, and time t. If canopy ice is less than 1E-3 then it is treated as not present.

$$Q_{sub} = \begin{cases} \left(\frac{C_{aice}}{\Delta t}\right) & \left(\frac{C_{aice}}{\Delta t}\right) < Q_{sub} \\ Q_{sub} & \left(\frac{C_{aice}}{\Delta t}\right) > Q_{sub} \end{cases} \tag{6-38}$$

$$C_{aice} = \begin{cases} 0 & 0 > C_{aice} + (R_{sint} - R_{sdrip})\Delta t + (Q_{frost} - Q_{sub})\Delta t \\ C_{aice} + (R_{sint} - R_{sdrip})\Delta t + (Q_{frost} - Q_{sub})\Delta t & 0 < C_{aice} + (R_{sint} - R_{sdrip})\Delta t + (Q_{frost} - Q_{sub})\Delta t \end{cases} \quad (6\text{-}39)$$

6.2.7 The Wetted Fraction of the Canopy

If canopy ice exists C_{aice}, then the fraction of the canopy that is wet F_{wet} depends on the maximum canopy capacity for snow interception M_{snow}.

$$F_{wet} = \begin{cases} 0 & 0 > \dfrac{C_{aice}}{\begin{cases} M_{snow} & M_{snow} > 1*10^{-6} \\ 1*10^{-6} & M_{snow} < 1*10^{-6} \end{cases}} \\ \dfrac{C_{aice}}{\begin{cases} M_{snow} & M_{snow} > 1*10^{-6} \\ 1*10^{-6} & M_{snow} < 1*10^{-6} \end{cases}} & 0 < \dfrac{C_{aice}}{\begin{cases} M_{snow} & M_{snow} > 1*10^{-6} \\ 1*10^{-6} & M_{snow} < 1*10^{-6} \end{cases}} \end{cases} \quad (6\text{-}40)$$

If no canopy ice exists then the fraction of the canopy that is wet is dependent on the canopy liquid C_{liq} and the maximum canopy capacity for liquid interception M_{liq}.

$$F_{wet} = \begin{cases} 0 & 0 > \dfrac{C_{liq}}{\begin{cases} M_{liq} & M_{liq} > 1*10^{-6} \\ 1*10^{-6} & M_{liq} < 1*10^{-6} \end{cases}} \\ \dfrac{C_{liq}}{\begin{cases} M_{liq} & M_{liq} > 1*10^{-6} \\ 1*10^{-6} & M_{liq} < 1*10^{-6} \end{cases}} & 0 < \dfrac{C_{liq}}{\begin{cases} M_{liq} & M_{liq} > 1*10^{-6} \\ 1*10^{-6} & M_{liq} < 1*10^{-6} \end{cases}} \end{cases} \quad (6\text{-}41)$$

$$F_{wet} = \begin{cases} F_{wet} & F_{wet} < 1 \\ 1 & F_{wet} > 1 \end{cases} * 0.667$$

(6-42)

6.2.8 Phase changes

If canopy ice exists according to the above qualifications, and vegetation temperature T_v is *greater* than freezing T_{frz}, then the following can be determined for the melting rate of snow within the canopy R_{melt}, the canopy ice C_{aice}, the canopy liquid C_{liq}, and the vegetation temperature.

$$R_{melt} = \begin{cases} \left(\dfrac{C_{aice}}{\Delta t}\right) & \left(\dfrac{C_{aice}}{\Delta t}\right) < (T_v - T_{frz})\left(\dfrac{\frac{C_{ice} * C_{aice}}{\rho_{ice}}}{L_f \Delta t}\right) \\ (T_v - T_{frz})\left(\dfrac{\frac{C_{ice} * C_{aice}}{\rho_{ice}}}{L_f \Delta t}\right) & \left(\dfrac{C_{aice}}{\Delta t}\right) > (T_v - T_{frz})\left(\dfrac{\frac{C_{ice} * C_{aice}}{\rho_{ice}}}{L_f \Delta t}\right) \end{cases} \quad (6\text{-}43)$$

$$C_{aice} = \begin{cases} 0 & 0 > \left(C_{aice} - ((C_{melt})\Delta t)\right) \\ \left(C_{aice} - ((C_{melt})\Delta t)\right) & 0 < \left(C_{aice} - ((C_{melt})\Delta t)\right) \end{cases} \quad (6\text{-}44)$$

$$C_{liq} = \begin{cases} 0 & 0 > \left(C_{liq} + ((C_{melt})\Delta t)\right) \\ \left(C_{liq} + ((C_{melt})\Delta t)\right) & 0 < \left(C_{liq} + ((C_{melt})\Delta t)\right) \end{cases} \quad (6\text{-}45)$$

$$T_v = (F_{wet} - T_{frz}) + (1 - F_{wet})T_v \quad (6\text{-}46)$$

If canopy liquid exists according to the above qualifications, and vegetation temperature is *greater* than freezing, then the following can be determined for the freezing rate of liquid water within the canopy, the canopy ice, the canopy liquid, and the vegetation temperature.

$$C_{frz} = \begin{cases} \left(\frac{C_{liq}}{\Delta t}\right) & \left(\frac{C_{liq}}{\Delta t}\right) < (T_v - T_{frz})\left(\frac{\frac{k_{w*}C_{liq}}{\rho_w}}{L_f\Delta t}\right) \\ (T_v - T_{frz})\left(\frac{\frac{k_{w*}C_{liq}}{\rho_w}}{L_f\Delta t}\right) & \left(\frac{C_{liq}}{\Delta t}\right) > (T_v - T_{frz})\left(\frac{\frac{k_{w*}C_{liq}}{\rho_w}}{L_f\Delta t}\right) \end{cases} \tag{6-47}$$

$$C_{aice} = \begin{cases} 0 & 0 > \left(C_{aice} + \left((C_{frz})\Delta t\right)\right) \\ \left(C_{aice} + \left((C_{frz})\Delta t\right)\right) & 0 < \left(C_{aice} + \left((C_{frz})\Delta t\right)\right) \end{cases} \tag{6-48}$$

$$C_{liq} = \begin{cases} 0 & 0 > \left(C_{liq} - \left((C_{frz})\Delta t\right)\right) \\ \left(C_{liq} - \left((C_{frz})\Delta t\right)\right) & 0 < \left(C_{liq} - \left((C_{frz})\Delta t\right)\right) \end{cases} \tag{6-49}$$

$$T_v = \left(F_{wet} - T_{frz}\right) + (1 - F_{wet})T_v \tag{6-50}$$

6.2.9 Total Canopy Water

The amount of intercepted water per ground area A_{wint} is determined by the sum of canopy intercepted liquid and canopy intercepted ice mass.

$$A_{wint} = C_{aice} + C_{liq} \tag{6-51}$$

6.2.10 Total Canopy Evaporation

The total canopy evaporation E_{can} is treated as the sum of evaporation R_{eva} and sublimation Q_{sub} subtracted from the difference of the dew R_{dew} and frost formation Q_{frost}.

$$E_{can} = R_{eva} + Q_{sub} - R_{dew} - R_{frost} \tag{6-52}$$

6.2.11 Rain or Snow on the Ground

Rain and snow on the ground (P_g and P_{sg} respectively) is the result of the through-fall, bulk density snowfall BD_{fall}, and the drip rate R_{drip} and R_{sdrip} . D_{zs} is snow depth increasing rate.

$$P_g = R_{drip} + R_{throu} \tag{6-53}$$

$$P_{sg} = R_{sdrip} + R_{sthrou} \tag{6-54}$$

$$D_{zs} = \frac{P_{sg}}{BD_{fall}} \tag{6-55}$$

If the surface is a lake (S_{typ} = 2) and the ground temperature T_g is above freezing T_{frz} then there is no snow at the ground and the snow depth D_{zs} is not increasing (both are set equal to zero).

6.3 Soil Water

For the case when snowmelt water is too large, for each soil layer the Effective porosity is:

$$\theta_e = \theta_c - \theta_{ice-soil} \tag{6-56}$$

The accumulation of the saturation excess is:

$$\theta_{total} = \sum_{i=1}^{nsoil} \max\left(0, \left(\theta_{soil}\right)_i - \theta_e\right) \times \left(d_{snow}\right)_i \tag{6-57}$$

where: *nsoil* is the number of the soils.
The soil liquid water content for each soil layer is set to:

$$\theta_{soil} = min(\theta_e, \theta_{soil}) \tag{6-58}$$

Impermeable fraction due to frozen soil for each soil layer is:

$$f_{cr} = \frac{max\left(0.0,\ e^{(-A\times(1-f_{ice}))} - e^{(-A)}\right)}{\left(1 - e^{(-A)}\right)} \tag{6-59}$$

where: A=4 and f_{ice} is the ice fraction in frozen soil and in the model defined as:

$$f_{ice} = \min\left(1.0, \frac{\theta_{ice\text{-}soil}}{\theta_c}\right) \tag{6-60}$$

In this equation $\theta_{ice\text{-}soil}$ is the soil ice moisture (m^3/m^3) and θ_c is the porosity, saturated value of soil moisture.

The maximum soil ice content and minimum liquid water of all layers are defined based on the following conditions:

$$\begin{cases} \theta_{ice-max} = \theta_{soil-ice} & if \quad \theta_{ice-soil} > \theta_{soil-max} \\ f_{cr} = f_{cr-max} & if \quad f_{cr} > f_{cr-max} \\ \theta_{soil} = \theta_{soil-min} & if \quad \theta_{soil-min} > \theta_{soil} \end{cases} \tag{6-61}$$

Subsurface runoff is calculated by the following equation:

$$R_{sb} = \left(1 - f_{cr-max}\right) \times f_{base} \times e^{(-Grid_{topo})} \times e^{\left(-K_{runoff} \times d_w\right)} \tag{6-62}$$

where: R_{sb} is the subsurface runoff, f_{cr-max} is he maximum impermeable fraction due to the frozen soil, d_w is the water table (section 7.2), f_{base} and K_{runoff} are the base flow coefficient and runoff decay factor respectively which are equal to 4 and 2.

different equations are defined in the model, to calculate surface runoff:

Case (I): in this case if the water input to the soil surface is greater than zero then $K_{runoff} = 6$ and surface runoff and infiltration rate are:

$$R_s = Q_{wat} \times \left[\left(1 - f_{cr}\right) \times f_{sat} + f_{cr}\right] \tag{6-63}$$

$$I_{sfc} = Q_{wat} - R_s \tag{6-64}$$

where: R_s is the surface runoff, Q_{wat} is the water input on the soil surface, I_{sfc} is the infiltration rate at surface, f_{sat} is the saturated fraction of the area and in this case is:

$$f_{sat} = f_{sat-\max} \times e^{\left(-0.5 \times K_{runoff} \times \left(d_w - 2\right)\right)} \tag{6-65}$$

Case (II): in this case if the water input to the soil surface is greater than zero the surface runoff and infiltration rate are determined by the same equation as the case (I). However, the $K_{runoff} = 2$ and f_{sat} is parameterized as:

$$f_{sat} = f_{sat-\max} \times e^{\left(-0.5 \times K_{runoff} \times d_w\right)} \quad (6\text{-}66)$$

Case (III): using subroutine INFIL (Section 7.3).
Case (IV): in this case the surface runoff and infiltration rate are determined by the same equation as the case (I), but f_{sat} is parameterized as:

$$f_{sat} = max\left(0.01, \frac{\theta_{2m-ave}}{\theta_c}\right)^4 \quad (6\text{-}67)$$

where: θ_{2m_ave} is 2-m averaged soil moisture (m^3/m^3), θ_c *is* porosity, saturated value of soil moisture (volumetric).

2-m averaged soil moisture is defined as:

$$\theta_{2m\_ave} = \frac{\sum_{i=1}^{nsoil} \left(\theta^N\right)_i \times \left(d_{snow}\right)_i}{d_{s-2m} = \sum_{i=1}^{nsoil} \left(d_{snow}\right)_i} \quad (6\text{-}68)$$

It should be noted that if the $d_{s\text{-}2m}$ is greater than 2m then sets as 2m.

In the model if the infiltration rate times time interval is bigger than Snow/soil layer thickness times saturated value of soil moisture, then the iteration time becomes twice

In this step, the accumulation of the saturation excess is calculated by:

$$\theta_{total} = \sum_{t=1}^{niter} \left(\theta_{sat}\right)_t \quad (6\text{-}69)$$

where: ***niter*** is number of iterations and θ_{sat} is the saturation excess of the total soil [m] (section 7.6) .

The total surface runoff converted from the (m/s) to (mm/s) by:

$$R_s = R_s \times 1000 + \frac{\theta_{total} \times 1000}{\Delta t} \quad (6\text{-}70)$$

Removal of soil water due to the groundwater flow (*R_{rm}*)

At each soil layer the amount of the removal of soil water is simulated as:

$$R_{rm} = R_s \times \Delta t \times \frac{K_i \times \left(d_{snow}\right)_i}{\sum_{i=1}^{nsoil} K_i \times \left(d_i\right)_{snow}} \quad (6\text{-}71)$$

Then the new soil liquid water content is calculated as:

$$\theta_{soil} = \theta_{soil}(Eq.7-3) - \frac{R_{rm}}{d_{snow} \times 1000} \tag{6-72}$$

In this model soil/snow liquid water mass (m_{snow}) should be equal or greater than minimum soil volume soil moisture ($m_{wat\text{-}min}$=0.1 mm). Otherwise water needed to bring m_{snow} from the lower layer. The m_{snow} for each soil layer expressed as:

$$m_{snow} = \theta_{soil} \times d_{snow} \times 1000 \tag{6-73}$$

The difference between m_{snow} and $m_{wat\text{-}min}$ should be subtracted from the subsurface runoff:

$$R_{sb} = R_{sb} - \frac{(m_{wat\text{-}min} - m_{snow})}{Dt} \tag{6-74}$$

After that the soil liquid water content should be recalculated using equation (7-18):

$$\theta_{soil} = \frac{m_{snow}}{d_{snow} \times 1000} \tag{6-75}$$

6.3.1 Water Table (Subroutine ZWTEQ, page 89)

The initial value of the water table is calculated by:

$$d_w = -3 \times Z_b - 0.001 \tag{6-76}$$

However for each fine soil layer of the 6m soil:

$$d_w = Z_{100} \qquad if \qquad abs(D_{\theta-100L} - D_{\theta-4L}) \le 0.1, \tag{6-77}$$

where: Z_{100} is layer-bottom depth of the *100-L* soil layers to 6.0 m. Which is equal to the number of the fine soil times layer thickness of the *100-L* soil layers to 6.0 m, $D_{\theta\text{-}4L}$ is water deficit from coarse (*4-L*) soil moisture profile and $D_{\theta\text{-}100L}$ is water deficit from fine (100-L) soil moisture profile respectively and are defined as:

$$D_{\theta-4L} = \sum_{i=1}^{nsoil} \left[\theta_c - (\theta_{soil})_i \times (d_{snow})_i \right], \tag{6-78}$$

$$D_{\theta-100L} = \sum_{i=1}^{N_{fine}} \left\{ \theta_c \times \left[1 - \left(1 + \frac{d_w - (Z_{100})_i}{m_{sat}} \right)^{-1/B} \right] \times DZ_{100} \right\}, \tag{6-79}$$

In the above equations d_{snow} is Snow/soil layer thickness and DZ_{100} is the layer thickness of the *100-L* soil layers to 6.0 m and defines as:

$$DZ_{100} = 3 \times (-Z_b) / N_{fine} \tag{6-80}$$

In which: Z_b is the depth of soil layer-bottom [m], and N_{fine} is number of fine soil layers of 6m soil.

6.3.2 Infiltration (Subroutine INFIL, page 89)

If the water input on soil surface is greater than zero (Q_{wat} >0) the time step is converted to the ratio of a day, therefore the new time is:

$$\Delta t_1 = \frac{\Delta t}{86400} \tag{6-81}$$

And the difference between saturated water content and permanent wilting point is:

$$\theta_{\text{c-wp}} = \theta_{\text{c}} - \theta_{\text{wp}} \tag{6-82}$$

In the first layer:

$$d_{ice} = -\left(Z_b\right)_{i=1} \times \left(\theta_{soil\text{-}ice}\right)_{i=1} \tag{6-83}$$

$$\left(d_{max}\right)_{i=1} = \left(-\left(Z_b\right)_{i=1} \times \theta_{c-wp}\right) \times \left(1 - \left(\left(\theta_{soil}\right)_{i=1} + \left(\theta_{soil-ice}\right)_{i=1} - \theta_{wp}\right) / \theta_{c-wp}\right) \tag{6-84}$$

For the layers 2 thru last layer (nsoil) :

$$d_{ice} = \sum_{i=2}^{nsoil} \left[d_{ice} - \left(\left(Z_b\right)_{i-1} - \left(Z_b\right)_i\right) \times \left(\theta_{\text{soil-ice}}\right)_i \right] \tag{6-85}$$

$$\left(d_{max}\right)_i = \left(\left(\left(Z_b\right)_{i-1} - \left(Z_b\right)_i\right) \times \theta_{c-wp}\right) \times \left(1 - \left(\left(\theta_{soil}\right)_i + \left(\theta_{soil-ice}\right)_i - \theta_{wp}\right) / \theta_{c-wp}\right) \tag{6-86}$$

$$d_{tot} = \sum_{i=1}^{nsoil} \left(d_{max}\right)_i \tag{6-87}$$

$$I_{max} = \frac{\left(P_x \times \left(\frac{d_{tot} \times \left(1 - e^{\left(-K_{dt} \times \Delta t_1\right)}\right)}{P_x + d_{tot} \times \left(1 - e^{\left(-K_{dt} \times \Delta t_1\right)}\right)} \right) \right)}{\Delta t} \tag{6-88}$$

Where: $\theta_{soil-ice}$ is soil ice moisture (m^3/m^3), θ_{soil} is soil liquid water (m^3/m^3), θ_{wp} is the wilting point soil moisture (volumetric), and $P_x = \max\left(0, Q_{wat} \times \Delta t\right)$,

Impermeable fraction due to frozen soil:

The impermeable fraction due to the frozen soil (f_{cr}) is expressed as:

$$\begin{cases} f_{cr} = 1 & \textit{default} \\ f_{cr} = 1 - T \times e^{\left(\frac{-3 \times FR_{data} \times F_{RZ}}{d_{ice}}\right)} & \textit{if} \quad d_{ice} > 0.01 \end{cases} \tag{6-89}$$

where: FR_{data} is used to compute maximum infiltration rate and F_{RZ} is:

$$F_{RZ} = \frac{\theta_c}{FC} \times \frac{0.412}{0.468} \tag{6-90}$$

And "***T"*** is:

FCR = 1.

```
IF (DICE > 1.E-2) THEN
ACRT = CVFRZ * FRZX / DICE
SUM = 1.
IALP1 = CVFRZ - 1
DO J = 1,IALP1
K = 1
DO JJ = J +1,IALP1
K = K * JJ
END DO
SUM = SUM + (ACRT ** (CVFRZ - J)) / FLOAT(K)
END DO
FCR = 1. - EXP (-ACRT) * SUM
END IF
```

Correction of infiltration limitation:

$$I_{max} = I_{max} \times f_{cr} \tag{6-91}$$

$$I_{max} = max\left(I_{max}, K\right) \tag{6-92}$$

$$I_{max} = min\left(I_{max} \times P_x\right) \tag{6-93}$$

Finally, the runoff and infiltration rate at the surface are:

$$R_s = max\left(0, Q_{wat} - I_{max}\right) \tag{6-94}$$

$$I_{sfc} = Q_{wat} - R_s \tag{6-95}$$

where: I_{max} is maximum infiltration, f_{cr} is impermeable fraction due to the frozen soil, K is hydraulic conductivity, R_s is surface runoff, I_{sfc} infiltration rate at the surface.

6.3.3 Calculate the right hand side of the time tendency term of the soil water diffusion equation. Also to compute (prepare) the matrix coefficients for the tri-diagonal matrix of the implicit time scheme (Subroutine SRT, page 91)

The soil water flux (q) is calculated by defining the following conditions:

$$\begin{cases} q = \lambda_i \times z_i + k_i - I_{sfc} + ET_i + E_{sfc} & i = 1 \\ q = \lambda_i \times z_i + k_i - \lambda_{i-1} \times z_{i-1} - k_{i-1} + ET_i & i < nsoil \\ q = \lambda_{i-1} \times z_{i-1} + k_{i-1} + ET_i + Q_{drain} & i = nsoil \end{cases} \tag{6-96}$$

where: λ is soil water diffusivity, Z is the height above some datum, I_{sfc} is the infiltration rate at surface, ET is transpiration rate. E_{sfc} is the soil surface evaporation rate, K is the hydraulic conductivity, Q_{drain} is soil bottom free drainage. In the above equations Z is:

$$\begin{cases} z_i = 2\times(\theta_i - \theta_{i+1}) / -(Z_b)_{i+1} & if \quad i=1 \\ z_i = 2\times(\theta_i - \theta_{i+1}) / \left[(Z_b)_{i+1} - (Z_b)_{i-1}\right] & if \quad i < nsoil \end{cases}, \tag{6-97}$$

Where: the θ for each soil layer is defined based on options for frozen soil permeability (*Opt_inf*). There are two options for frozen soil permeability, one is linear effects, more permeable (Niu and Yang, 2006, JHM)and second is nonlinear effects, less permeable (old):

$$\begin{cases} \theta_i = \theta^N & if \quad Opt\_inf = 1 \\ \theta_i = \theta_{soil} & if \quad Opt\_inf = 2 \end{cases}, \tag{6-98}$$

Also, based on the options for runoff and groundwater (*Opt_run*), Q_{drain} is calculated as:

$$\begin{cases} Q_{drain} = 0 & Opt\_run = 1 or 2 \\ Q_{drain} = S \times K_i & Opt\_run = 3 \\ Q_{drain} = (1 - F_{cr-\max}) \times K_i & Opt\_run = 4 \end{cases}, \tag{6-99}$$

Options for runoff and groundwater are included:

1: TOPMODEL with groundwater (Niu et al. 2007 JGR).
2: TOPMODEL with an equilibrium water table (Niu et al. 2005 JGR).
3: original surface and subsurface runoff (free drainage).
4: BATS surface and subsurface runoff (free drainage)

In the above equation "*S*" is the slope index (0-1)

The matrix coefficients for the tri-diagonal matrix are classified to the three groups:
For the first soil layer:

$$\begin{cases} AI = 0.0 \\ BI = \lambda \times 2 / \left[(-Z_b)_{i+1} \times (-Z_b)_i\right], \\ CI = -BI \end{cases} \tag{6-100}$$

If $i<nsoil$

$$\begin{cases} AI_i = -\lambda_{i-1} \times 2 / \left[(-Z_b)_{i-1} \times (-Z_b)_i \times (-Z_b)_i\right] \\ CI = -\lambda_i \times 2 / \left[(-Z_b)_{i+1} \times (-Z_b)_i \times (-Z_b)_i\right] \\ BI = -(CI + AI) \end{cases}, \tag{6-101}$$

If $i=nsoil$ (last layer)

$$\begin{cases} AI_i = -\lambda_{i-1} \times 2 / \left[(-Z_b)_{i-1} \times (-Z_b)_i \times (-Z_b)_i\right] \\ CI = 0 \\ BI = -(CI + AI) \end{cases}, \tag{6-102}$$

For all soil layers the RHS (right hand side of the matrix) is calculated by:

$$\begin{cases} RHS = q / Z_b \\ RHS = -q / \left(\left(Z_b \right)_{i-1} - \left(Z_b \right)_i \right) \end{cases}, \tag{6-103}$$

6.3.3.1 Soil water diffusivity and hydraulic conductivity (Subroutine WDFCND1, page 94)

Soil water diffusivity and hydraulic conductivity are expressed as:

$$\lambda = \left[\lambda_{sat} \times \left[max\left(0.01, \theta^N / \theta_c \right) \right]^{B+2} \right] \times \left(1 - f_{cr}\right), \tag{6-104}$$

$$K = \left[K_{sat} \times \left[max\left(0.01, \theta^N / \theta_c \right) \right]^{2 \times B+3} \right] \times \left(1 - f_{cr}\right), \tag{6-105}$$

6.3.3.2 Soil water diffusivity and hydraulic conductivity (Subroutine WDFCND2, page 94)

$$\lambda = \lambda_{sat} \times \left[max\left(0.01, \theta^N / \theta_c \right) \right]^{B+2}, \tag{6-106}$$

However if the soil ice moisture is greater than zero $\left(\theta_{soil-ice} > 0\right)$ the hydraulic conductivity is updated as:

$$\begin{aligned} \lambda = & \left[1 / \left(1 + \left(500 \times \theta_{soil-ice} \right)^3 \right) \right] \times \left[\lambda_{sat} \times \left[max\left(0.01, \theta^N / \theta_c \right) \right]^{B+2} \right] + \\ & \left[1 - 1 / \left(1 + \left(500 \times \theta_{soil-ice} \right)^3 \right) \right] \times \lambda_{sat} \times \left(2 / \theta_c \right)^{(2+B)} \end{aligned}, \tag{6-107}$$

$$K = K_{sat} \times \left[max\left(0.01, \theta^N / \theta_c \right) \right]^{2 \times B+3}, \tag{6-108}$$

Where: the λ_{sat} is saturated soil hydraulic diffusivity, θ^N is total soil water content (m^3/m^3). θ_C is effective porosity, and B depends on soil texture.

6.3.4 Calculate/Update soil moisture content values (Subroutine SSTEP, page 92)

This subroutine updates the matrix coefficients to solve the saturation excess water for each soil layer:

$$\begin{cases} RHS_i = RHS_i \times \Delta t \\ AI_i = AI_i \times \Delta t \\ BI_i = BI_i \times \Delta t \\ CI_i = CI_i \times \Delta t \end{cases}, \tag{6-109}$$

After solving the matrix coefficients (Section 7.5.1) the soil liquid water content (θ_{sat}) is updated for all layers as:

$$\left(\theta_{sat}\right)_i = \left(\theta_{sat}\right)_i + CI_i \tag{6-110}$$

Excessive water above saturation in a layer is moved to its unsaturated layer like in a bucket

$$\theta_{sat} = max\left[\left\{(\theta_{soil})_i - (\theta_c) + (\theta_{soil-ice})_i\right\}, 0.0\right] \times (d_{snow})_i \quad \text{(6-111}$$

$$\theta_{soil} = min\left[\left\{(\theta_c) + (\theta_{soil-ice})_i\right\}, (\theta_{soil})_i\right] \quad \text{(6-112)}$$

$$(\theta_{sat})_i = \sum_{i=nsoil}^{2}\left[\theta_c / (d_{snow})_{i-1}\right] \quad \text{(6-113)}$$

For the first layer:

$$\theta_{sat} = max\left[\left\{(\theta_{soil})_1 - (\theta_c) + (\theta_{soil-ice})_1\right\}, 0.0\right] \times (d_{snow})_1 \quad \text{(6-114}$$

$$\theta_{soil} = min\left[\left\{(\theta_c) + (\theta_{soil-ice})_1\right\}, (\theta_{soil})_1\right] \quad \text{(6-115)}$$

6.3.4.1 Solves (invert) the matrix coefficients show bellow (Subroutine ROSR12):

```
B(1),  C(1),   0  ,   0  ,   0  ,  . . . ,     0    # #      #   #      #
A(2),  B(2), C(2),   0  ,   0  ,  . . . ,     0    # #      #   #      #
 0  ,  A(3), B(3), C(3),   0  ,  . . . ,     0    # #      #   # D(3) #
 0  ,   0  , A(4), B(4), C(4),  . . . ,     0    # # P(4) #   # D(4) #
 0  ,   0  ,  0  , A(5), B(5),  . . . ,     0    # # P(5) #   # D(5) #
 .                              .                # #   .  # = #   .  #
 .                              .                # #   .  #   #   .  #
 .                              .                # #   .  #   #   .  #
 0  , . . . ,  0  ,A(M-2), B(M-2), C(M-2),  0    # #P(M-2)#   #D(M-2)#
 0  , . . . ,  0  ,  0   , A(M-1), B(M-1), C(M-1)# #P(M-1)#   #D(M-1)#
 0  , . . . ,  0  ,   0   ,   0   ,  A(M) ,  B(M) # # P(M) #   # D(M) #
```

Initialize equation coefficient C for the lowest soil layer.

$$\begin{aligned} C_{nsoil} &= 0.0 \\ P_{ntop} &= -C_{ntop} / B_{ntop} \end{aligned} \quad \text{(6-116)}$$

Where: the subscript "*ntop*" and "*nsoil*" refer to the first and last layers.
Solve the equation coefficients for the first soil layer.

$$\Delta_{ntop} = D_{ntop} / B_{ntop} \quad \text{(6-117)}$$

Solve the equation coefficients for soil layers 2 thru last layer

$$\begin{aligned} P_i &= -C_i \times \left(\frac{1}{B_i} + A_i \times P_{i-1}\right) \\ \Delta_i &= (D_i - A_i \times \Delta_{i-1}) \times \left(\frac{1}{B_i + A_i \times P_{i-1}}\right) \end{aligned} \quad \text{(6-118)}$$

Set P to Δ for the lowest layer

$$P_{nsoil} = \Delta_{nsoil} \tag{6-119}$$

Adjust P for soil layers 2 thru last layer

$$P_{ii} = P_{ii} \times P_{ii+1} + \Delta_{ii} \tag{6-120}$$

Where: "*ii*" is:

$$ii = nsoil - i + (ntop - 1) + 1 \tag{6-121}$$

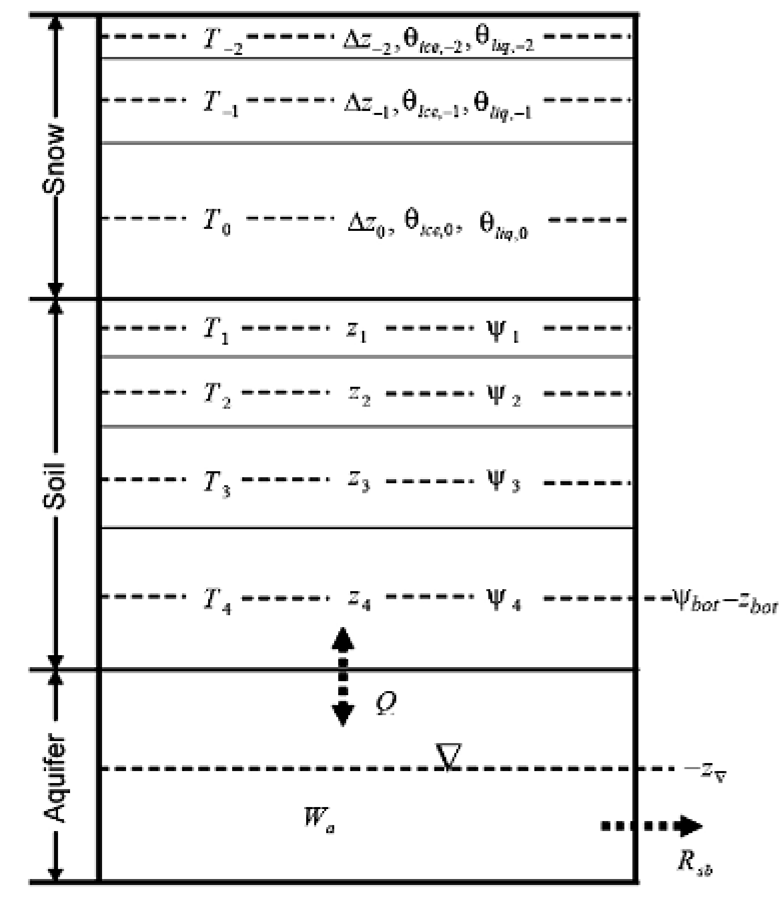
```
DO K = NTOP+1,NSOIL

KK = NSOIL - K + (NTOP-1) + 1          ?????????????????  it could be        NSOIL - K +1
P (KK) = P (KK) * P (KK +1) + DELTA (KK)

END DO
```

6.4 SIMGM groundwater model

The groundwater model within Noah-MP (OPT_RUN=1) is the SIMGM model (Niu et al. 2007). The basics of this model are described as follows.



As shown in the figure, an unconfined aquifer is defined as the part below soil column in the Noah-MP model. The dynamics of the water storage in the aquifer (W_a) could be expressed as

$$\frac{dW_a}{dt} = Q - R_{sb}$$

Where Q is recharge to the aquifer, and R_{sb} is discharge from the aquifer.

The recharge is calculated based on Darcy's law

$$Q = -K_a \frac{-z_\nabla - (\psi_{bot} - z_{bot})}{z_\nabla - z_{bot}}$$

where K_a is aquifer hydraulic conductivity, z_∇ is water table depth, ψ_{bot} is matric potential of the bottom layer, and z_{bot} is the node depth of the bottom layer.

The discharge (i.e. subsurface runoff) is parameterized as

$$R_{sb} = \left(1 - f_{frz,\max}\right) \bullet R_{sb,\max} \bullet e^{-\lambda} \bullet e^{-f\left(z_{\nabla} - 2\right)}$$

where $f_{frz,\max}$ is the maximum impermeable fraction due to frozen soil, $R_{sb,\max}$ is the base flow coefficient, and λ (=10.5) is grid cell mean topographic index.

Each time the SIMGM model is called, W_a, z_{∇}, and ϑ（soil liquid water） are updated, according to the location of z_{∇}. If z_{∇} is below the bottom of soil column, z_{∇} is updated as

$$z_{\nabla} = \left(-z_{i=4} + 25\right) -_a W \big/ \left(1000 \times 0.2\right),$$

liquid water mass is also updated $m_{liq} = m_{liq} - Q \bullet dt$.

If z_{∇} is within the 4th soil layer,

$$z_{\nabla} = -z_{i=4} - \left(W_a - 0.2 \times 1000 \times 25\right) \big/ \left(1000 Poro_{e,i=4}\right)$$

where $Poro_{e,i=4}$ is the effective porosity at the 4th soil layer.

$$m_{liq} = m_{liq} - R_{sb} K_a \Delta z \bullet dt \big/ WT_{sub}$$

where Δz is the layer thickness, and $WT_{sub} = \sum_{i=1}^{4} K_{a,i} \Delta z_i$.

Finally update the soil liquid water $\vartheta = m_{liq} \big/ \Delta z$ for each layer.

References

Niu, G.-Y., Z.-L. Yang, R.E. Dickinson, and L.E. Gulden, 2005: A simple TOPMODEL-based runoff parameterization (SIMTOP) for use in global climate models, *Journal of Geophysical Research,* 110, D21106, doi:10.1029/2005JD006111.

Niu, G.-Y. and Z.-L. Yang, 2006: Effects of frozen soil on snowmelt runoff and soil water storage at a continental scale, *Journal of Hydrometeorology,* 7 (5), 937-952.

Niu, G.-Y., Z.-L. Yang, R. E. Dickinson, L. E. Gulden, and H. Su, 2007: Development of a simple groundwater model for use in climate models and evaluation with Gravity Recovery and Climate Experiment data, *J. Geophys. Res.*, 112, D07103, doi:10.1029/2006JD007522.

7. Dynamic Vegetation

The respiration reduction factor $r_f = \begin{cases} 0.5 & non-growing\ season \\ 1 & growing\ season \end{cases}$, while the growing season is determined by phenology subroutine.

The temperature factor $T_f = ARM^{\frac{T_v - 298.16}{10}}$,

leaf respiration $Resp = RMF_{25} \bullet T_f \bullet Fnf \bullet LAI \bullet r_f \bullet (1 - w_{stress})$

where :

Leaf maintenance respiration per time step $rs_{leaf} = \min\left\{ m_{leaf} / \Delta t,\ Resp \bullet 12 \times 10^{-6} \right\}$

Find root respiration per time step $rs_{root} = RMR_{25} \bullet m_{root} \bullet 10^{-3} \bullet T_f \bullet r_f \bullet 12 \bullet 10^{-6}$

Stem respiration $rs_{stem} = RMS_{25} \bullet m_{stem} \bullet 10^{-3} \bullet T_f \bullet r_f \bullet 12 \bullet 10^{-6}$

Wood respiration $rs_{wood} = RS_{wood,c} \bullet r \bullet m_{wood} \bullet P_{wood}$

Then convert the carbon assimilation from $\mu\,\mathrm{mol}\,\mathrm{CO_2} / m^2 / s$ to $g\ carbon / m^2 / s$

The carbon flux assimilated per time step $F_{carbon} = PSN \bullet 12 \bullet 10^{-6}$

The fraction of carbon flux goes into leaf $f_{c,leaf} = \exp\left\{ 0.01 \bullet \left[1 - \exp\left(0.75 LAI \right) \right] \bullet LAI \right\}$

except when VEGTYP is 12, 0.75 in the above equation is replaced by 0.50.

The fraction of carbon flux goes into stem $f_{c,stem} = LAI / 10$

Then update $f_{c,leaf}$ by reduction of $f_{c,stem}$ $f_{c,leaf} = f_{c,leaf} - f_{c,stem}$.

The wood to root ratio

$$f_{wood} = \begin{cases} \left(1 - e^{-BF \bullet WRRAT \bullet m_{root}/m_{wood}} / BF\right) \bullet P_{wood} & m_{wood} > 0 \\ 0 & m_{wood} < 0 \end{cases}$$

The fraction of carbon flux goes into root $f_{c,stem} = \left(1 - f_{c,leaf}\right) \bullet \left(1 - f_{wood}\right)$

and the fraction of carbon flux goes into wood $f_{c,wood} = \left(1 - f_{c,leaf}\right) f_{wood}$

Next, calculate the leaf and root turnover at each time step:

$$Ovt_{leaf} = C_{ovt,leaf} \bullet 10^{-6} \bullet m_{leaf}$$

$$Ovt_{stem} = C_{ovt,stem} \bullet 10^{-6} \bullet m_{stem}$$

$$Ovt_{root} = C_{ovt,root} \bullet m_{root}$$

$$Ovt_{wood} = 9.5 \bullet 10^{-10} \bullet m_{leaf}$$

Then calculate seasonal leaf dying rate based on temperature and water stress. Water stress is set to 1 at permanent wilting point.

$$SC = \exp\left[-0.3 \bullet \max\left(0, T_v - T_{d,leaf}\right)\right] \bullet m_{leaf} \ /120$$

$$SD = \exp\left(w_{stress} - 1\right) \bullet C_{wstress}$$

$$Dy_{leaf} = m_{leaf} \bullet 10^{-6} W_{dy,leaf} \bullet SD + SC \bullet C_{dy,leaf}$$

$$Dy_{stem} = m_{stem} \bullet 10^{-6} W_{dy,leaf} \bullet SD + SC \bullet C_{dy,leaf}$$

Calculate the growth respiration for leaf, stem, root and wood.

$$gr_{leaf} = \max\left\{0,\ f_{gr} \bullet \left(f_{c,leaf} \bullet f_{carbon} - rs_{leaf} \right)\right\}$$

$$gr_{stem} = \max\left\{0,\ f_{gr} \bullet \left(f_{c,stem} \bullet f_{carbon} - rs_{stem} \right)\right\}$$

$$gr_{root} = \max\left\{0,\ f_{gr} \bullet \left(f_{c,root} \bullet f_{carbon} - rs_{root} \right)\right\}$$

$$gr_{wood} = \max\left\{0,\ f_{gr} \bullet \left(f_{c,wood} \bullet f_{carbon} - rs_{wood} \right)\right\}$$

Limit lower T limit for photosynthesis, and then update leaf, stem overturn through adding some limits to avoid reducing the mass below its minimum value.

Net primary productivities

$$NPP_{leaf} = \max\left\{\Delta NPP_{leaf}, -del_{leaf}\right\}$$

$$NPP_{stem} = \max\left\{\Delta NPP_{stem}, -del_{stem}\right\}$$

$$NPP_{root} = f_{c,root} \bullet F_{carbon} - rs_{root} - gr_{root}$$

$$NPP_{wood} = f_{c,wood} \bullet F_{carbon} - rs_{wood} - gr_{wood}$$

Update the masses

$$m_{leaf} = m_{leaf} + \left(NPP_{leaf} - Ovt_{leaf} - dy_{leaf} \right) \bullet \Delta t$$

$$m_{stem} = m_{stem} + \left(NPP_{stem} - Ovt_{stem} - dy_{stem} \right) \bullet \Delta t$$

$$m_{root} = m_{root} + \left(NPP_{root} - Ovt_{root} \right) \bullet \Delta t$$

$$m_{wood} = \left[m_{wood} + \left(NPP_{wood} - Ovt_{wood} \right) \bullet \Delta t \right] \bullet P_{wood}$$

Calculate soil carbon budgets:

Short lived carbon pool $P_{c,fast} = P_{c,fast} + \left(Ovt_{root} + Ovt_{leaf} + Ovt_{stem} + Ovt_{wood} + dy_{leaf} \right) \bullet \Delta t$

Soil temperature factor for microbial respiration $fs_T = 2^{\left(T_{soil,i=1} - 283.16/10 \right)}$

Soil water factor for microbial respiration $fs_w = \dfrac{0.23 w_{root}}{\left(0.2 + w_{root} \right)\left(0.23 + w_{root} \right)}$

Soil respiration per time step $rs_{soil} = 12 \bullet 10^{-6} fs_w \bullet fs_T \bullet MRP \bullet \max\left\{ 0, P_{c,fast} \bullet 10^{-3} \right\}$

Then update $P_{c,fast} = P_{c,fast} - 1.1 rs_{soil} \bullet \Delta t$

and stable carbon pool $P_{c,stable} = P_{c,stable} + 0.1 rs_{soil} \bullet \Delta t$

Finally, the net carbon flux from land to the atmosphere

$$F_{c,net} = -F_{carbon} + rs_{leaf} + rs_{root} + rs_{wood} + rs_{root} + rs_{soil} + gr_{leaf} + gr_{root} + gr_{wood}$$

and $LAI = \max\left\{ m_{leaf} \bullet LAPM, LAI_{\min} \right\}$

$$SAI = \max\left\{ m_{leaf} \bullet SAPM, SAI_{\min} \right\}$$