Rock Physics

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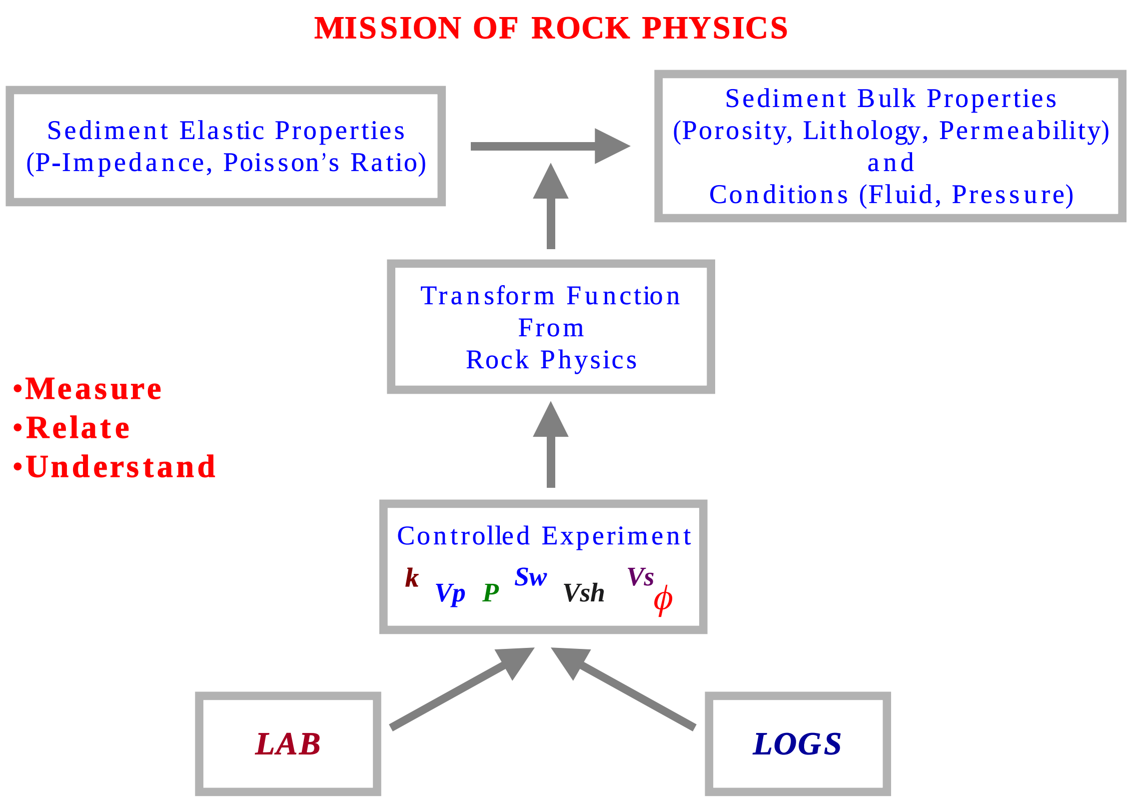
Preface

Interpretation of Seismic Data. The main geophysical tool for illuminating the subsurface is seismic. Seismic data yield a map of the elastic properties of the subsurface. This map is useful as long as it can be interpreted to delineate structures and, most important, quantify reservoir properties. Rock physics provides links between the sediment's elastic properties and its bulk properties (porosity, lithology) and conditions (pore pressure and pore fluid).

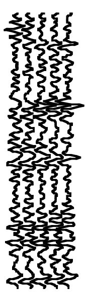
What is Rational Rock Physics. Rock physics' mission is to translate seismic observables into reservoir properties, e.g., translate impedance into porosity. The simplest approach is to compile a laboratory data set, relevant to the site under investigation, where, e.g., impedance and porosity are measured on a set samples. The resulting impedance-porosity trend can be applied to seismic impedance to map it into porosity. The applicability of an empirical trend is as good as the data set it has been derived from. Extrapolation outside of the data set range is possible only if the physics is understood and theoretically generalized.

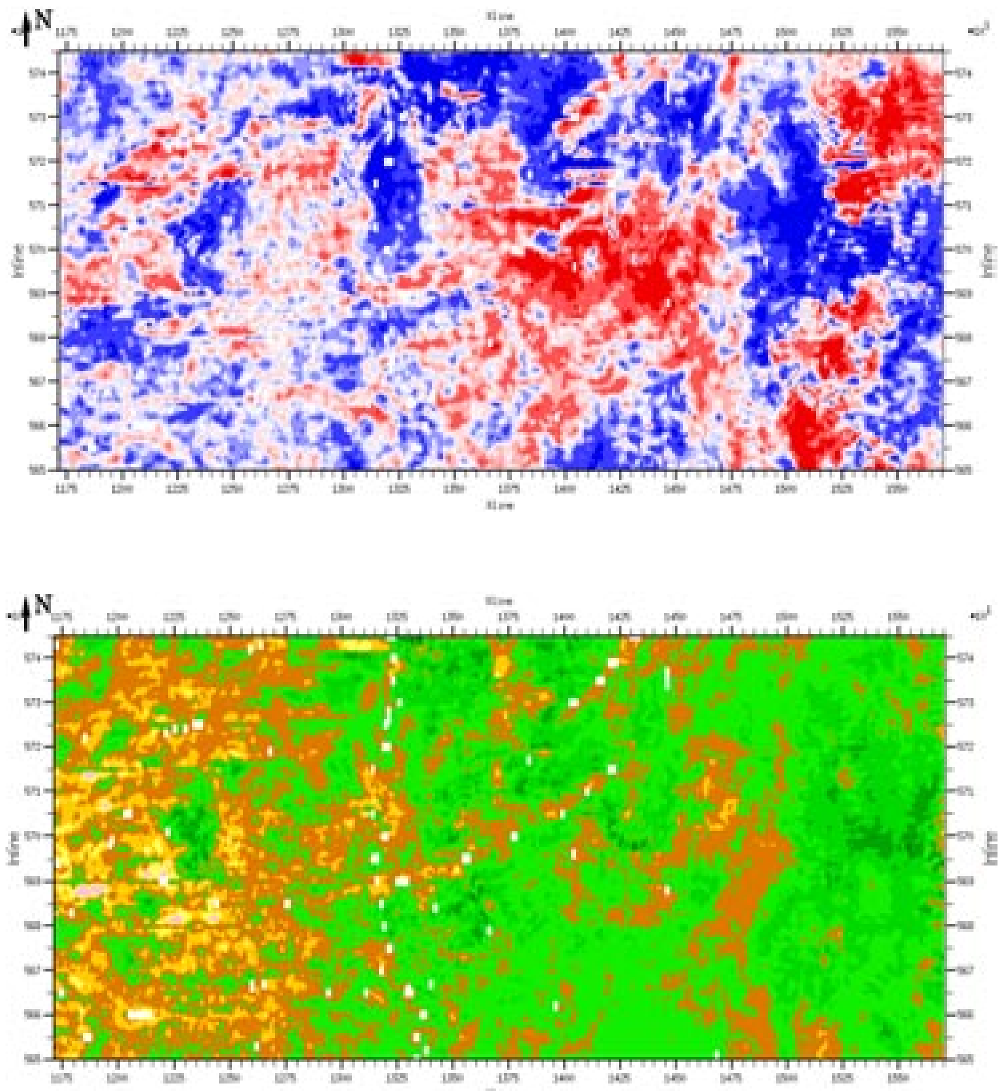


Methods of Rock Physics
Reflection and Inversion

Reflection amplitude carries information about elastic contrast in the subsurface. Inversion attempts to translate this information into elastic properties at a point in space.

Point properties are important because we are interested in absolute values of porosity and saturation at a point in space.



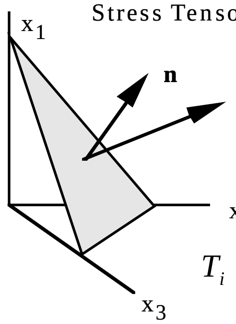


La Cira Norte. Courtesy Ecopetrol and Mario Gutierrez.

Basics

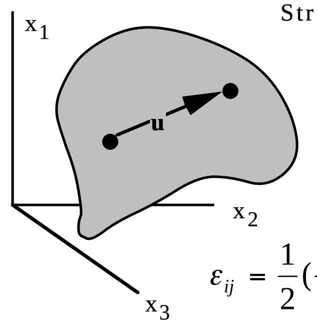
Elasticity

Stress Tensor



$$T_i = \sigma_{ij} n_j$$

Strain Tensor



$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

$$\sigma_{ij} = \sigma_{ji}\ i \neq j;\ \varepsilon_{ij} = \varepsilon_{ji}\ i \neq j.$$

Hooke's law: 21 independent constants

$$\sigma_{ij} = c_{ijkl} e_{kl};\ c_{ijkl} = c_{jikl} = c_{ijlk} = c_{jilk},\ c_{ijkl} = c_{klij}.$$

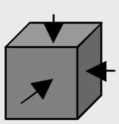
Isotropic Hooke's law: 2 independent constants (elastic moduli)

$$\sigma_{ij} = \lambda \delta_{ij} \varepsilon_{\alpha\alpha} + 2\mu \varepsilon_{ij};\ \varepsilon_{ij} = [(1+\nu)\sigma_{ij} - \nu \delta_{ij} \sigma_{\alpha\alpha}] / E.$$

λ and μ -- Lame's constants; ν -- Poisson's ratio; E -- Young's modulus.

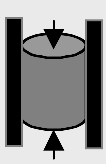
Bulk Modulus

$$K = \lambda + 2\mu / 3$$



Compressional Modulus

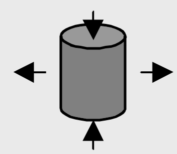
$$M = \lambda + 2\mu$$

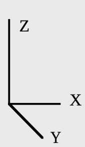


Young's Modulus and Poisson's Ratio

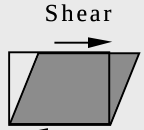
$$E = \mu(3\lambda + 2\mu) / (\lambda + \mu)$$

$$\nu = 0.5\lambda / (\lambda + \mu)$$





Shear



$$\sigma_{xz} = 2\mu \varepsilon_{xz}$$

$$\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{zz} = \varepsilon_{xy} = 0$$

Young

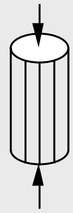
$$\sigma_{zz} = E\varepsilon_{zz}$$

$$\nu = -\varepsilon_{xx} / \varepsilon_{zz}$$

$$\sigma_{xx} = \sigma_{yy} =$$

$$\sigma_{xy} = \sigma_{xz} =$$

$$\sigma_{yz} = 0$$



Compressional

$$\sigma_{zz} = M\varepsilon_{zz}$$

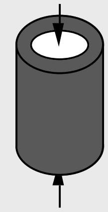
$$\sigma_{yy} = \lambda \varepsilon_{zz} =$$

$$[\lambda / (\lambda + 2\mu)]\sigma_{zz} =$$

$$[\nu / (1 - \nu)]\sigma_{zz}$$

$$\varepsilon_{xx} = \varepsilon_{yy} =$$

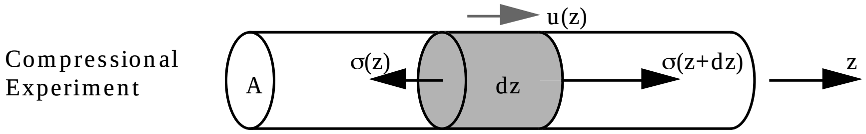
$$\varepsilon_{xy} = \varepsilon_{xz} = \varepsilon_{yz} = 0$$



Basics
Dynamic and Static Elasticity

WAVE EQUATION

$\varepsilon \sim 10^{-7}$



$$A dz \rho \ddot{u} = A[\sigma(z+dz) - \sigma(z)] = A dz \partial\sigma / \partial z \Rightarrow \rho \partial^2 u / \partial t^2 = \partial\sigma / \partial z$$

$$\sigma = M\varepsilon = M \partial u / \partial z \Rightarrow \partial^2 u / \partial t^2 = (M/\rho)\partial^2 u / \partial z^2 \equiv V_p^2 \partial^2 u / \partial z^2$$

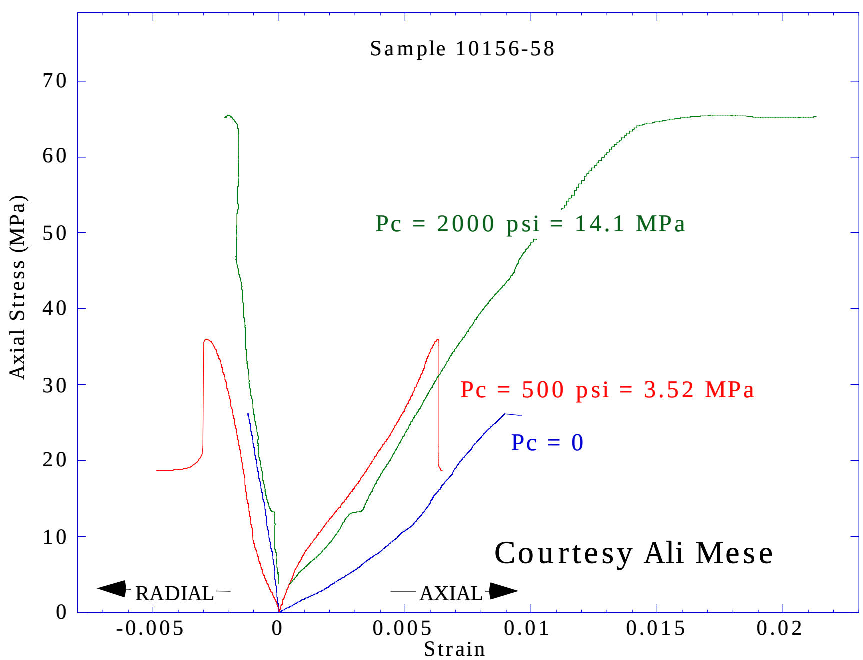
Dynamic definitions:

$$V_p = \sqrt{M/\rho} = \sqrt{(K + 4G/3)/\rho};\ V_s = \sqrt{G/\rho};$$

$$M = \rho V_p^2;\ G = \rho V_s^2;\ K = \rho(V_p^2 - 4V_s^2/3);\ \lambda = \rho(V_p^2 - 2V_s^2)$$

STATIC UNIAXIAL EXPERIMENT

$\varepsilon \sim 10^{-2}$



Velocity and Saturation
Gassmann's Equations

In static (low-frequency) limit, pore fluid affects only the bulk modulus of rock

Gassmann's Equations -- Basis of Fluid Substitution

$$\frac{K_{Sat}}{K_s - K_{Sat}} = \frac{K_{Dry}}{K_s - K_{Dry}} + \frac{K_f}{\phi(K_s - K_f)}$$

- K_{Sat}: Bulk Modulus of Rock w/Fluid
- K_s: Bulk Modulus of Mineral Phase
- K_{Dry}: Bulk Modulus of Dry Rock
- K_f: Bulk Modulus of Pore Fluid
- ϕ: Porosity

$$G_{Sat} = G_{Dry}$$

- G_{Sat}: Shear Modulus of Rock w/Fluid
- G_{Dry}: Shear Modulus of Dry Rock

The bulk modulus of rock saturated with a fluid is related to the bulk modulus of the dry rock and vice versa

$$K_{Sat} = K_s \frac{\phi K_{Dry} - (1+\phi) K_f K_{Dry} / K_s + K_f}{(1-\phi) K_f + \phi K_s - K_f K_{Dry} / K_s}$$

$$K_{Dry} = K_s \frac{1 - (1-\phi) K_{Sat} / K_s - \phi K_{Sat} / K_f}{1 + \phi - \phi K_s / K_f - K_{Sat} / K_s}$$

Velocity depends on the elastic moduli and density

$$V_p = \sqrt{(K_{Sat} + \frac{4}{3} G_{Dry}) / \rho_{Sat}}$$

$$V_s = \sqrt{G_{Dry} / \rho_{Sat}}$$

$$\rho_{Sat} = \rho_{Dry} + \phi \rho_{Fluid} > \rho_{Dry}$$

Velocity and Porosity
Summary of Theories

Rock Physics Models: Velocity-Porosity

Wyllie et al. (1956) Time Average (Empirical)

Recommended Parameters	
Rock Type	Vsolid (km/s)
Sandstone	5.480 to 5.950
Limestone	6.400 to 7.000
Dolomite	7.000 to 7.925

$$\frac{1}{V_p} = \frac{\phi}{V_{Fluid}} + \frac{1-\phi}{V_{Solid}}$$

Total Porosity (ϕ); P-wave Velocity (V_p); Sound Speed In Pore Fluid (V_{Fluid}); P-wave Velocity in Solid Phase (V_{Solid})

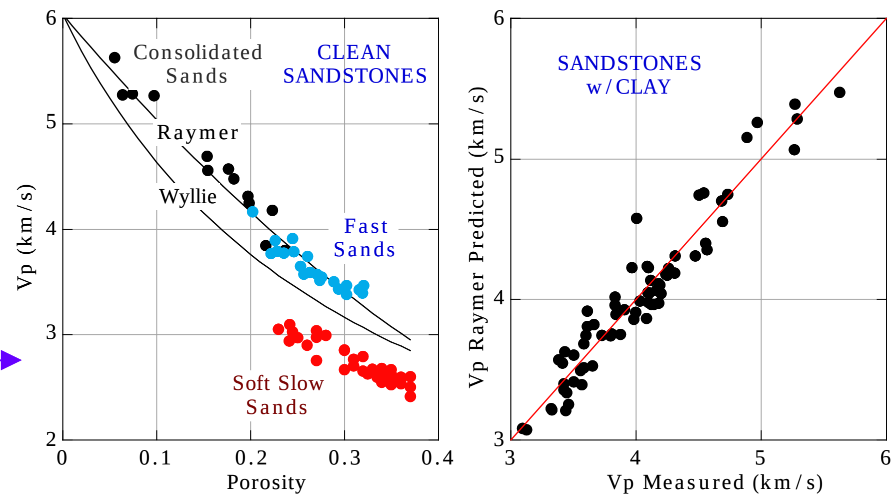
Raymer et al. (1980) Equations (Empirical)

$$V_p = (1-\phi)^2 V_{Solid} + \phi V_{Fluid} \Leftarrow \phi < 0.37$$

Raymer's equation is more accurate than Wyllie's.

Neither one of the two should be used to model soft slow sands.

Examples of applying Wyllie's and Raymer's equation to sandstone lab data →



Han (1986) Equations for Consolidated Sandstones (Empirical, Ultrasonic Lab Measurements)

Pressure	Saturation	Equations		Comments
40 MPa	100% Water	Vp=6.08-8.06ϕ	Vs=4.06-6.28ϕ	Clean Rock
40 MPa	100% Water	Vp=5.59-6.93ϕ-2.18C	Vs=3.52-4.91ϕ-1.89C	Rock w/Clay
30 MPa	100% Water	Vp=5.55-6.96ϕ-2.18C	Vs=3.47-4.84ϕ-1.87C	Rock w/Clay
20 MPa	100% Water	Vp=5.49-6.94ϕ-2.17C	Vs=3.39-4.73ϕ-1.81C	Rock w/Clay
10 MPa	100% Water	Vp=5.39-7.08ϕ-2.13C	Vs=3.29-4.73ϕ-1.74C	Rock w/Clay
5 MPa	100% Water	Vp=5.26-7.08ϕ-2.02C	Vs=3.16-4.77ϕ-1.64C	Rock w/Clay
40 MPa	Room-Dry	Vp=5.41-6.35ϕ-2.87C	Vs=3.57-4.57ϕ-1.83C	Rock w/Clay

Vp and *Vs* are in km/s; the total porosity ϕ is in fractions; volumetric clay content in the whole rock (not in the solid phase) C is in fractions.

Tosaya (1982) Equations for Shaley Sandstones (Empirical, Ultrasonic Lab Measurements)

Pressure	Saturation	Equations		Comments
40 MPa	100% Water	$Vp = 5.8 - 8.6\phi - 2.4C$	$Vs = 3.7 - 6.3\phi - 2.1C$	Rock w/Clay

Vp and *Vs* are in km/s; the total porosity ϕ is in fractions; volumetric clay content in the whole rock (not in the solid phase) C is in fractions.

Eberhart-Phillips (1989) Equations for Shaley Sandstones (Empirical, Based on Han's Data)

100% Water Saturation

$$V_p = 5.77 - 6.94\phi - 1.73\sqrt{C} + 0.446[P - \exp(-16.7P)]$$

$$V_s = 3.70 - 4.94\phi - 1.57\sqrt{C} + 0.361[P - \exp(-16.7P)]$$

Vp and *Vs* are in km/s; differential pressure *P* is in kilobars. 1 kb = 100 MPa.

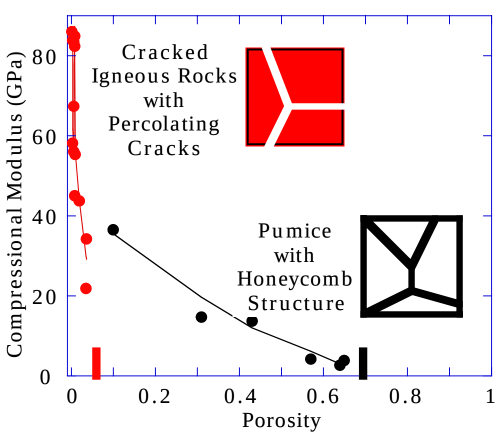
Nur's (1998) Critical Porosity Concept (Heuristic)

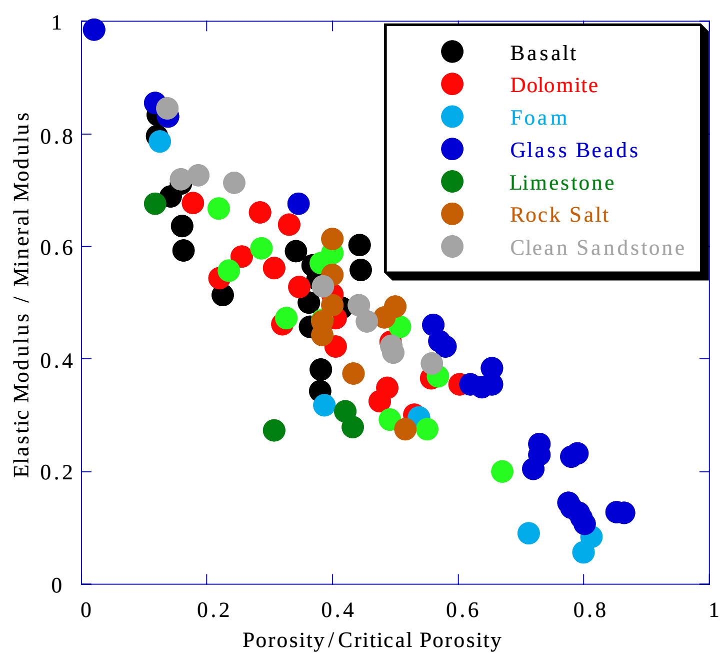
$$K_{Dry} = K_{Solid}(1 - \phi / \phi_c) \qquad G_{Dry} = G_{Solid}(1 - \phi / \phi_c)$$

K_{Dry}: Dry-Rock Bulk Modulus; K_{Solid}: Solid-Phase Bulk Modulus; ϕ: Total Porosity; G_{Dry}: Dry-Rock Shear Modulus; G_{Solid}: Solid-Phase Shear Modulus; ϕ_c: Critical Porosity

Critical Porosity of Various Rocks

Sandstones	36% - 40%
Limestones	36% - 40%
Dolomites	36% - 40%
Pumice	80%
Chalks	55% - 65%
Rock Salt	36% - 40%
Cracked Igneous Rocks	3% - 6%
Oceanic Basalts	20%
Sintered Glass Beads	36% - 40%
Glass Foam	85% - 90%





Examples of using Critical Porosity Concept to mimic lab data

Contact Cement Model (Theoretical)

Coordination Number — Critical Porosity — Cement's Compressional Modulus — Cement's Shear Modulus

$$K_{Dry} = \frac{n(1-\phi_c)M_c S_n}{6} \qquad G_{Dry} = \frac{3K_{Dry}}{5} + \frac{3n(1-\phi_c)G_c S_\tau}{20}$$

Dry-Rock Bulk Modulus — Dry-Rock Shear Modulus

$$S_n = A_n(\Lambda_n)\alpha^2 + B_n(\Lambda_n)\alpha + C_n(\Lambda_n),\ A_n(\Lambda_n) = -0.024153\cdot\Lambda_n^{-1.3646},$$

$$B_n(\Lambda_n) = 0.20405\cdot\Lambda_n^{-0.89008},\ C_n(\Lambda_n) = 0.00024649\cdot\Lambda_n^{-1.9864};$$

$$S_\tau = A_\tau(\Lambda_\tau,\nu_s)\alpha^2 + B_\tau(\Lambda_\tau,\nu_s)\alpha + C_\tau(\Lambda_\tau,\nu_s),$$

$$A_\tau(\Lambda_\tau,\nu_s) = -10^{-2}\cdot(2.26\nu_s^2 + 2.07\nu_s + 2.3)\cdot\Lambda_\tau^{0.079\nu_s^2+0.1754\nu_s-1.342},$$

$$B_\tau(\Lambda_\tau,\nu_s) = (0.0573\nu_s^2 + 0.0937\nu_s + 0.202)\cdot\Lambda_\tau^{0.0274\nu_s^2+0.0529\nu_s-0.8765},$$

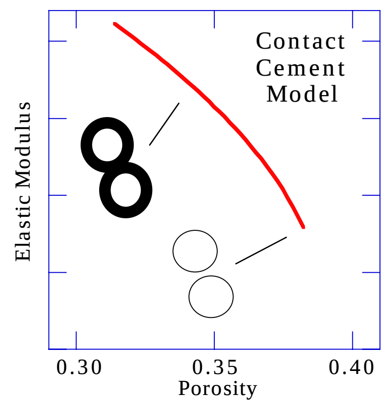
$$C_\tau(\Lambda_\tau,\nu_s) = 10^{-4}\cdot(9.654\nu_s^2 + 4.945\nu_s + 3.1)\cdot\Lambda_\tau^{0.01867\nu_s^2+0.4011\nu_s-1.8186};$$

$$\Lambda_n = 2G_c(1-\nu_s)(1-\nu_c)/[\pi G_s(1-2\nu_c)],\ \Lambda_\tau = G_c/(\pi G_s);$$

$$\alpha = [(2/3)(\phi_c-\phi)/(1-\phi_c)]^{0.5};$$

$$\nu_c = 0.5(K_c/G_c - 2/3)/(K_c/G_c + 1/3);$$

$$\nu_s = 0.5(K_s/G_s - 2/3)/(K_s/G_s + 1/3).$$

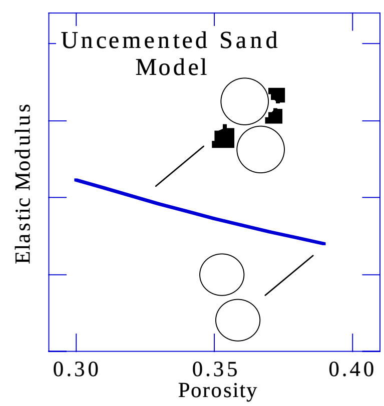


Uncemented Sand Model or Modified Lower Hashin-Shtrikman (Theoretical)

$$K_{Dry} = \left[\frac{\phi/\phi_c}{K_{HM}+\frac{4}{3}G_{HM}} + \frac{1-\phi/\phi_c}{K+\frac{4}{3}G_{HM}}\right]^{-1} - \frac{4}{3}G_{HM},$$

$$G_{Dry} = \left[\frac{\phi/\phi_c}{G_{HM}+z} + \frac{1-\phi/\phi_c}{G+z}\right]^{-1} - z,\ z = \frac{G_{HM}}{6}\left(\frac{9K_{HM}+8G_{HM}}{K_{HM}+2G_{HM}}\right);$$

$$K_{HM} = \left[\frac{n^2(1-\phi_c)^2G^2}{18\pi^2(1-\nu)^2}P\right]^{\frac{1}{3}},\ G_{HM} = \frac{5-4\nu}{5(2-\nu)}\left[\frac{3n^2(1-\phi_c)^2G^2}{2\pi^2(1-\nu)^2}P\right]^{\frac{1}{3}}.$$



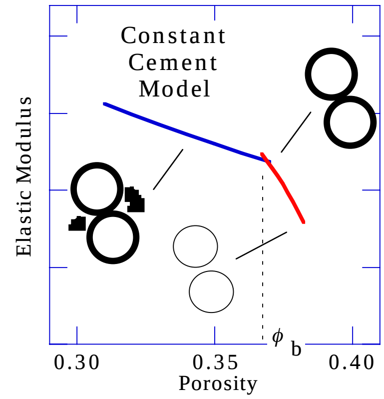
In the above equations, K stands for bulk modulus and G stands for shear modulus. ν is Poisson's ratio. Subscript "c" with a modulus means "cement" and subscript "s" means grain material. ϕ is the total porosity, and ϕ_c is critical porosity. P is differential pressure. All units have to be consistent.

Constant Cement Model (Theoretical)

$$K_{dry} = \left(\frac{\phi / \phi_b}{K_b + 4G_b / 3} + \frac{1 - \phi / \phi_b}{K_s + 4G_b / 3}\right)^{-1} - 4G_b / 3,$$

$$G_{dry} = \left(\frac{\phi / \phi_b}{G_b + z} + \frac{1 - \phi / \phi_b}{G_s + z}\right)^{-1} - z, \; z = \frac{G_b}{6}\frac{9K_b + 8G_b}{K_b + 2G_b}.$$

ϕ_b is porosity (smaller than ϕ_c) at which contact cement trend turns into constant cement trend. Elastic moduli with subscript "b" are the moduli at porosity ϕ_b. These moduli are calculated from the contact cement theory with $\phi = \phi_b$.

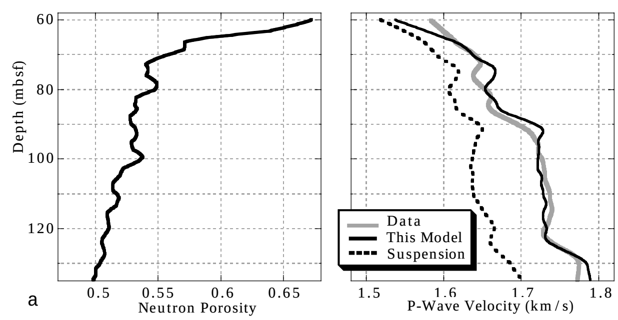


Model for Marine Sediments (Theoretical)

$$K_{Dry} = \left[\frac{(1-\phi)/(1-\phi_c)}{K_{HM} + \frac{4}{3}G_{HM}} + \frac{(\phi-\phi_c)/(1-\phi_c)}{\frac{4}{3}G_{HM}}\right]^{-1} - \frac{4}{3}G_{HM},$$

$$G_{Dry} = \left[\frac{(1-\phi)/(1-\phi_c)}{G_{HM} + z} + \frac{(\phi-\phi_c)/(1-\phi_c)}{z}\right]^{-1} - z,$$

$$z = \frac{G_{HM}}{6}\left(\frac{9K_{HM} + 8G_{HM}}{K_{HM} + 2G_{HM}}\right); \quad \phi > \phi_c.$$

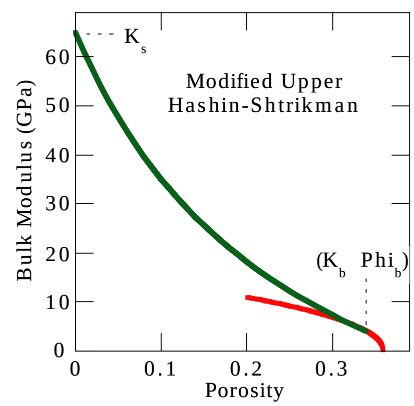


Consolidated Sand Model or Modified Upper Hashin-Shtrikman (Theoretical)

$$K_{Dry} = \left[\frac{\phi / \phi_b}{K_b + \frac{4}{3}G_s} + \frac{1 - \phi / \phi_b}{K_s + \frac{4}{3}G_s}\right]^{-1} - \frac{4}{3}G_s,$$

$$G_{Dry} = \left[\frac{\phi / \phi_b}{G_b + z} + \frac{1 - \phi / \phi_b}{G_s + z}\right]^{-1} - z, \quad z = \frac{G_s}{6}\frac{9K_s + 8G_s}{K_s + 2G_s}.$$

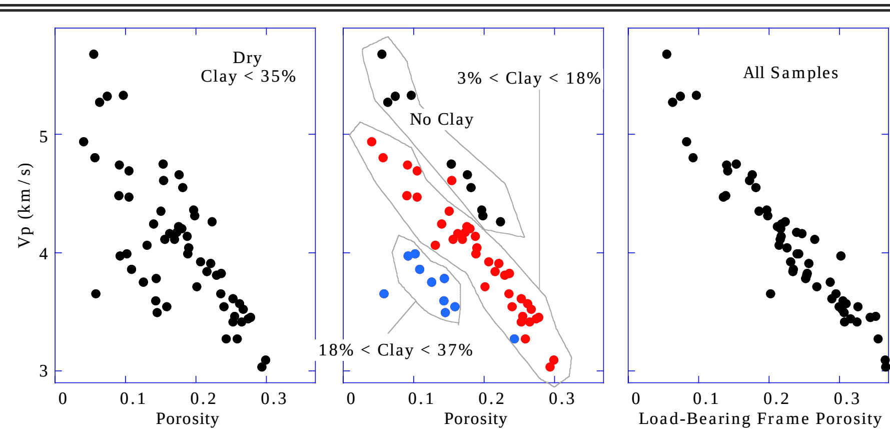
Elastic moduli with subscript "b" are the moduli at porosity ϕ_b. These moduli can be calculated from the contact cement theory with $\phi = \phi_b$, or chosen at some initial point as suggested by data.



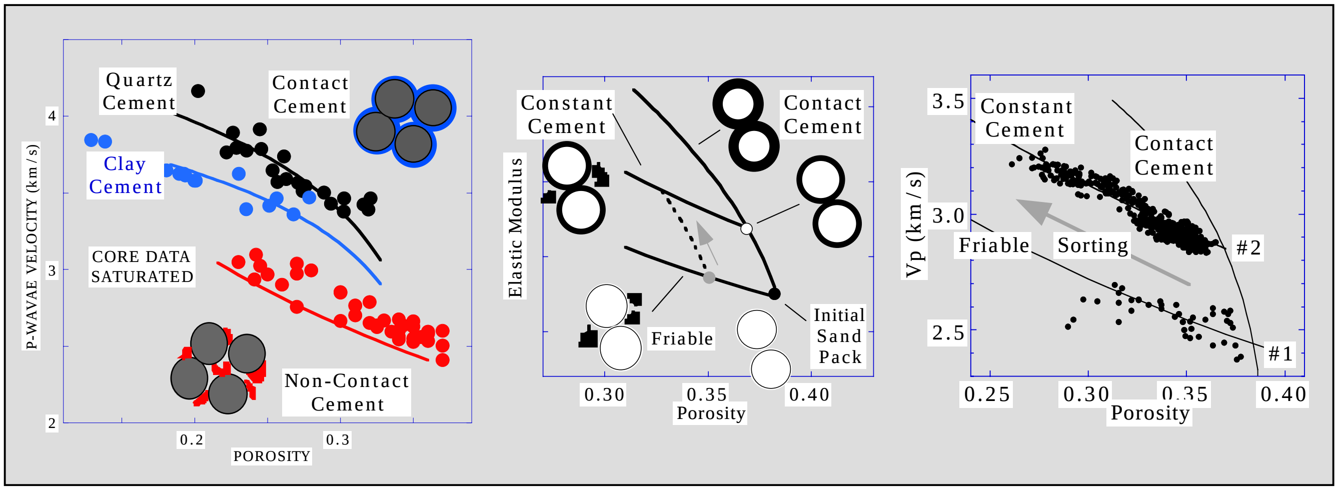
Non-Load-Bearing Clay Model (Theoretical)

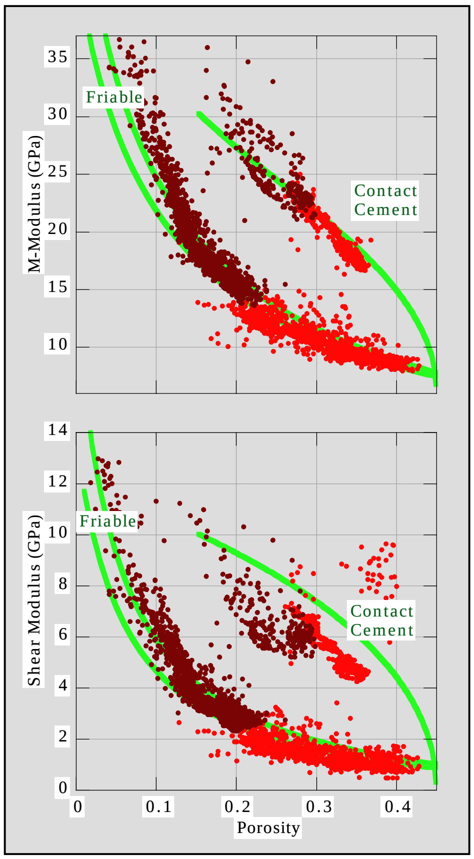
The velocity (or elastic moduli) are plotted versus the load-bearing frame porosity $\phi_F = \phi_t + C(1- \phi_{clay})$ instead of total porosity ϕ_t. C is the volume of clay in rock, and ϕ_{clay} is the internal porosity of clay.

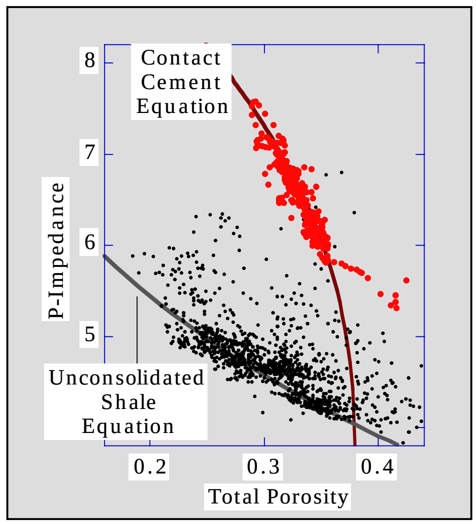
A scatter collapses onto a single trend (right frame).

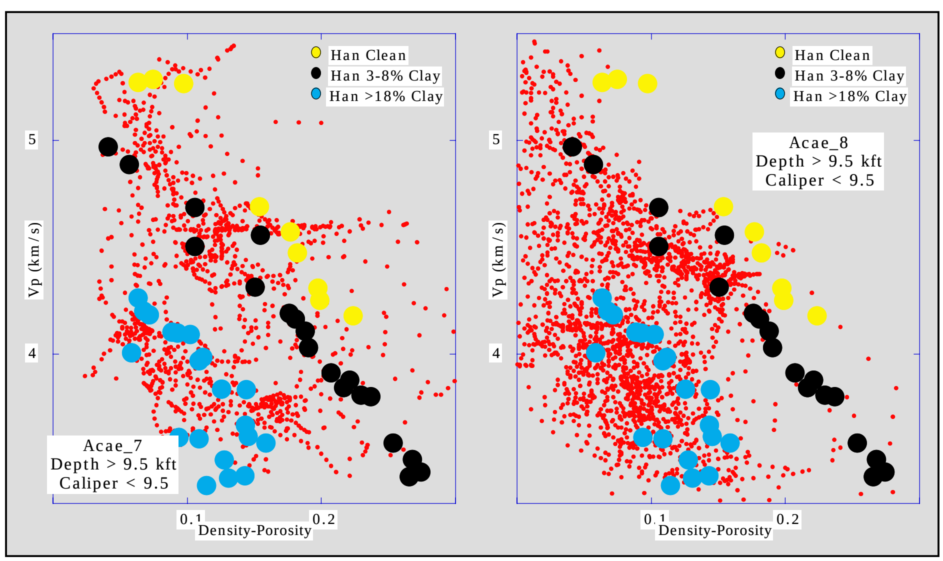


Examples: Velocity-Porosity





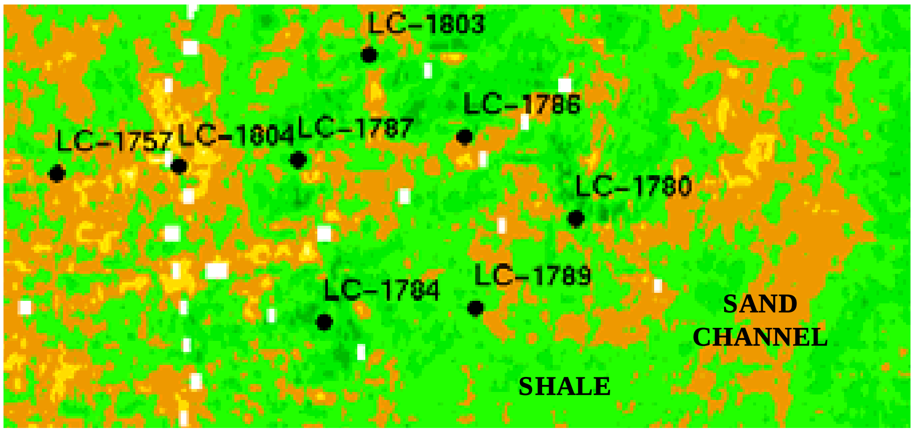


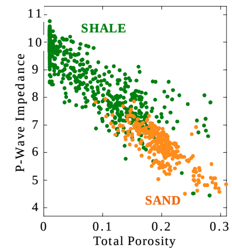


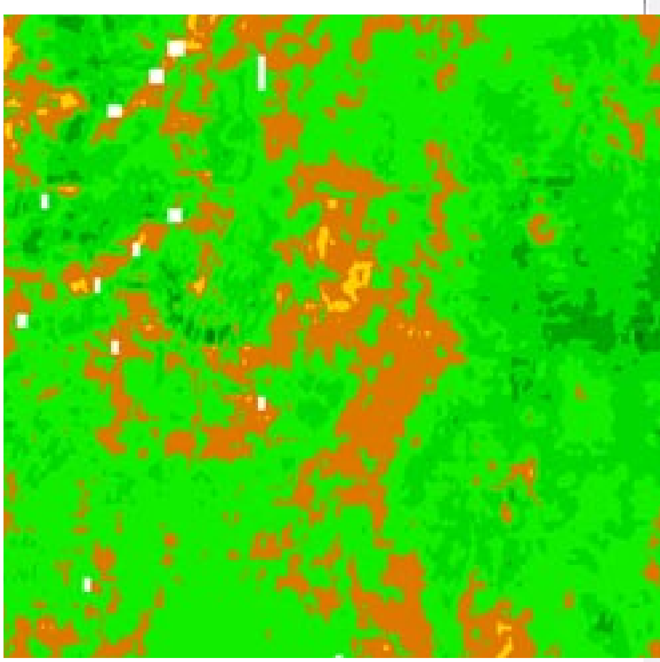
Velocity and Porosity

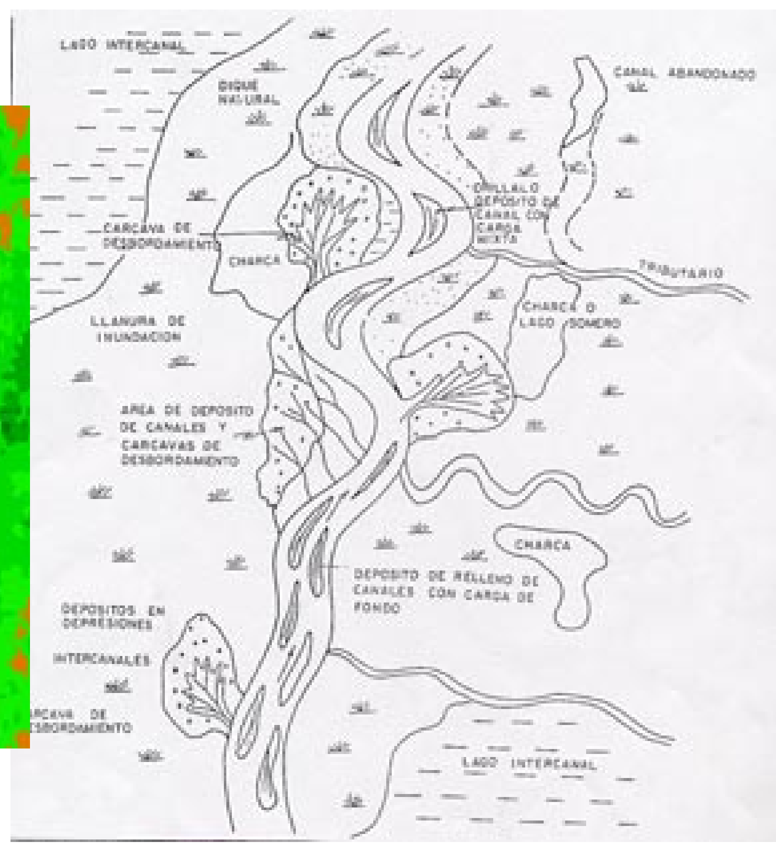
Fluvial Sandstones -- La Cira Case Study

Interpreting Impedance Inversion









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Vp and Vs Summary of Theories

Rock Physics Models: Vp and Vs

Castagna et al. (1985) Mudrock -- Empirical
Sand/Shale -- S_W = 100%

S-wave Velocity (km/s) P-wave Velocity (km/s)

$$V_S = 0.862 \cdot V_P - 1.172$$

Castagna et al. (1993) -- Empirical
Sand/Shale -- S_W = 100%

S-wave Velocity (km/s) P-wave Velocity (km/s)

$$V_S = 0.804 \cdot V_P - 0.856$$

Krief et al. (1990) -- "Critical Porosity'

Any mineral

Any fluid

$$\frac{V_{P\_Saturated}^2 - V_{Fluid}^2}{V_{S\_Saturated}^2} = \frac{V_{P\_Mineral}^2 - V_{Fluid}^2}{V_{S\_Mineral}^2}$$

Grinberg/Castagna (1992)

Empirical

Any mineral

S_W = 100%

S-wave Velocity (km/s) P-wave Velocity (km/s)

$$V_S = \frac{1}{2}\{[\sum_{i=1}^{4} X_i \sum_{j=0}^{2} a_{ij} V_P^j] + [\sum_{i=1}^{4} X_i (\sum_{j=0}^{2} a_{ij} V_P^j)^{-1}]^{-1}\}$$

$$\sum_{i=1}^{4} X_i = 1$$

i	Mineral	*ai2*	*ai1*	*ai0*
1	Sandstone	0	0.80416	-0.85588
2	Limestone	-0.05508	1.01677	-1.03049
3	Dolomite	0	o.58321	-0.07775
4	Shale	0	0.76969	0.86735

Willimas (1990) -- Empirical
S_W = 100%

Sand

Shale

S-wave Velocity (km/s) P-wave Velocity (km/s)

$$V_S = 0.846 \cdot V_P - 1.088$$

$$V_S = 0.784 \cdot V_P - 0.893$$