HW-3D: CALCULATING LEAST PRINCIPAL STRESSES

ANSWERS

GENERAL INPUT:

Basin: depth 5 Km

Pore water: hydrostatic gradient: 10 MPa/km

Overpressured gradient: 15 MPa/km

Sediments: Overburden gradient: 20 MPa/km

Poisson's ratio v'=0.25

Calculation of overburden and pore pressure profiles

The total vertical stress is calculated by intergrading the overburden: $\sigma_v = \rho g z$

Depth z = 0: σ_v = 0; z = 5km: σ_v = 100MPa

The hydrostatic pore pressure profile would be: $u_0 = \rho_w g z$

Depth z = 0: u_0 = 0; z = 5km: u_0 = 50MPa

However, the basin is overpressured with a gradient of 1.5gr/cc, therefore: u = 1.5gz

Depth z = 0: u = 0; z = 5km: u = 75MPa

Finally, the effective vertical stress can be calculated by subtracting the pore pressures from the overburden: $\sigma'_v = \sigma_v - u$

Depth z = 0: σ'_v = 0; z = 5km: σ'_v = 25MPa

A) CONSTANT RATIO BETWEEN HORIZONTAL AND VERTICAL EFFECTIVE STRESSES

1. Eaton (1969) calculated the ratio of horizontal to vertical stresses using concepts of the elastic theory:

$$\sigma_h = \left[\frac{v}{1-v}\right](\sigma_v - u) + u \qquad \text{Equation A.1}$$

where σ'_h is the effective horizontal stress, σ'_v the effective vertical stress and u the pore pressure.

Using $v = 0.25$ and the values for the vertical stress and pore pressure calculated above, the horizontal stress profiles are:

a. Horizontal total stress

$$\sigma_h = \left[\frac{1}{3}\right](\sigma_v - u) + u$$

Depth z = 0: $\sigma_h = 0$; z = 5km: σ_h = (1/3)x(100-75)+75 = 83.33 MPa

b. Horizontal effective stress

$$\sigma'_h = \left[\frac{1}{3}\right]\sigma'_v$$

Depth z = 0: $\sigma'_h = 0$; z = 5km: σ'_h = (1/3)x25= 8.33 MPa

Note that $\sigma_h = \sigma'_h + u$

These stress profiles are plotted in Figure A. The minimum principal stress is horizontal.

2. Zoback & Healy (1984) proposed a ratio of horizontal to vertical effective stresses based on the assumption that stresses in the Earth cannot exceed the frictional strength of pre-existing faults:

$$\sigma_3 = \left[\frac{1-\sin\phi}{1+\sin\phi}\right](\sigma_1 - u) + u \qquad \text{Equation A.2}$$

Where ϕ is the friction angle ($\mu = \tan(\phi)$), σ_1 the maximum principal stress and σ_3 the minimum principal stress.

Generally, following deposition, the horizontal stress is lower than the vertical stress, and therefore the minimum principal stress is horizontal. In other words, $\sigma_1 = \sigma_v$ and $\sigma_3 = \sigma_h$.

Using $\phi = 30$and the values for the vertical stress and pore pressure calculated above, the horizontal stress profiles are:

a. Horizontal total stress

$$\sigma_h = [0.333](\sigma_v - u) + u$$

Depth z = 0: σ_h = 0; z = 5km: σ_h = (0.333)x(100-75)+75 = 83.33 MPa

b. Horizontal effective stress

$$\sigma'_h = [0.333]\sigma'_v$$

Depth z = 0: σ'_h = 0; z = 5km: σ'_h = (0.333)x25= 8.33 MPa

Note that $\sigma_h = \sigma'_h + u$

These stress profiles are plotted in Figure A. The minimum principal stress is horizontal.

Note that both approaches calculate the same stress ratio. This is a result of the values we chose for Poisson's ratio and the friction angle. However it illustrates that these methods are approximations and have historically been used to fit existing data; they do not reflect the true behavior of the sediments during burial (see part 2.A of the homework).

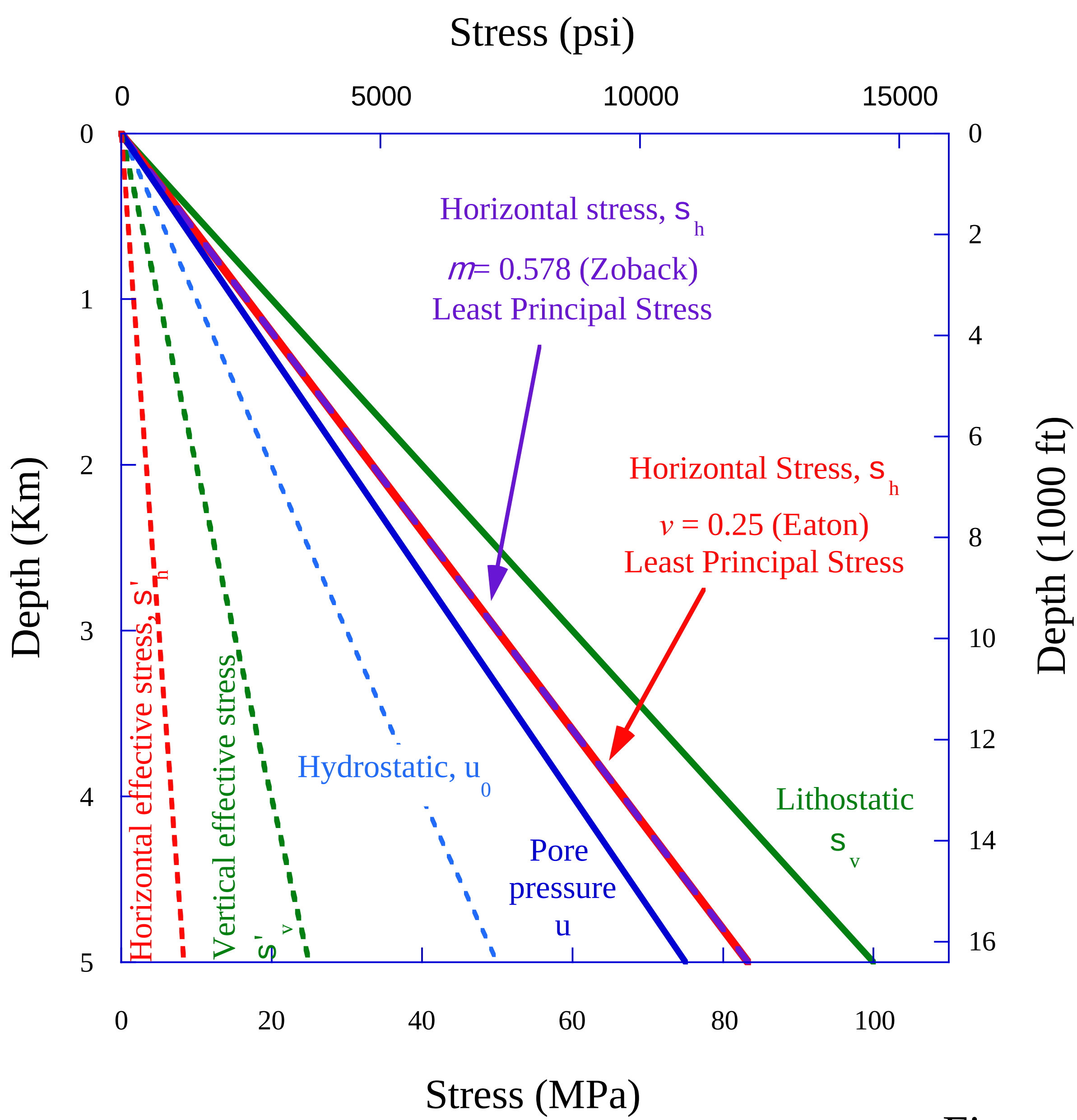
Stress (psi)
0
5000
10000
15000
Depth (Km)
0
1
2
3
4
5
Depth (1000 ft)
0
2
4
6
8
10
12
14
16
Horizontal stress, s h
m= 0.578 (Zoback)
Least Principal Stress
Horizontal Stress, s h
v = 0.25 (Eaton)
Least Principal Stress
Horizontal effective stress, s' h
Vertical effective stress
s' v
Hydrostatic, u 0
Pore
pressure
u
Lithostatic
s v
0
20
40
60
80
100
Stress (MPa)

Figure A

B) Stress ratio varying with depth

Various authors have proposed empirical curves that relate the stress ratio with depth (e.g. Eaton (1969), Matthews & Kelly (1967)).

Table B provides values for the stress ratio K_i based on the empirical relation proposed by Matthews & Kelly (1967) from the Louisiana Gulf Coast.

Depth (km)	K_i	σ_v (MPa)	u (MPa)	σ'_h (MPa)	σ_h (MPa)
1	0.43	20	15	2.15	17.15
2	0.56	40	30	5.6	35.6
3	0.67	60	45	10.05	55.05
4	0.78	80	60	15.6	75.6
5	0.85	100	75	21.25	96.25

Table B: Stress ratio K_i with depth from Matthews & Kelly (1967)

We calculate the vertical stress by $\sigma_v = \rho g z$ (as in the initial section of the solution) and the pore pressure by $u = 1.5gz$

Then, $\sigma_h = K_i(\sigma_v - u) + u$ Equation B.1

Figure B plots the calculated stress values with depth. The figure also plots the constant ratio solution for comparison. Note that the stress rises with depth and approaches the overburden. Up to the depth of 5 km, the least principal stress remains horizontal.

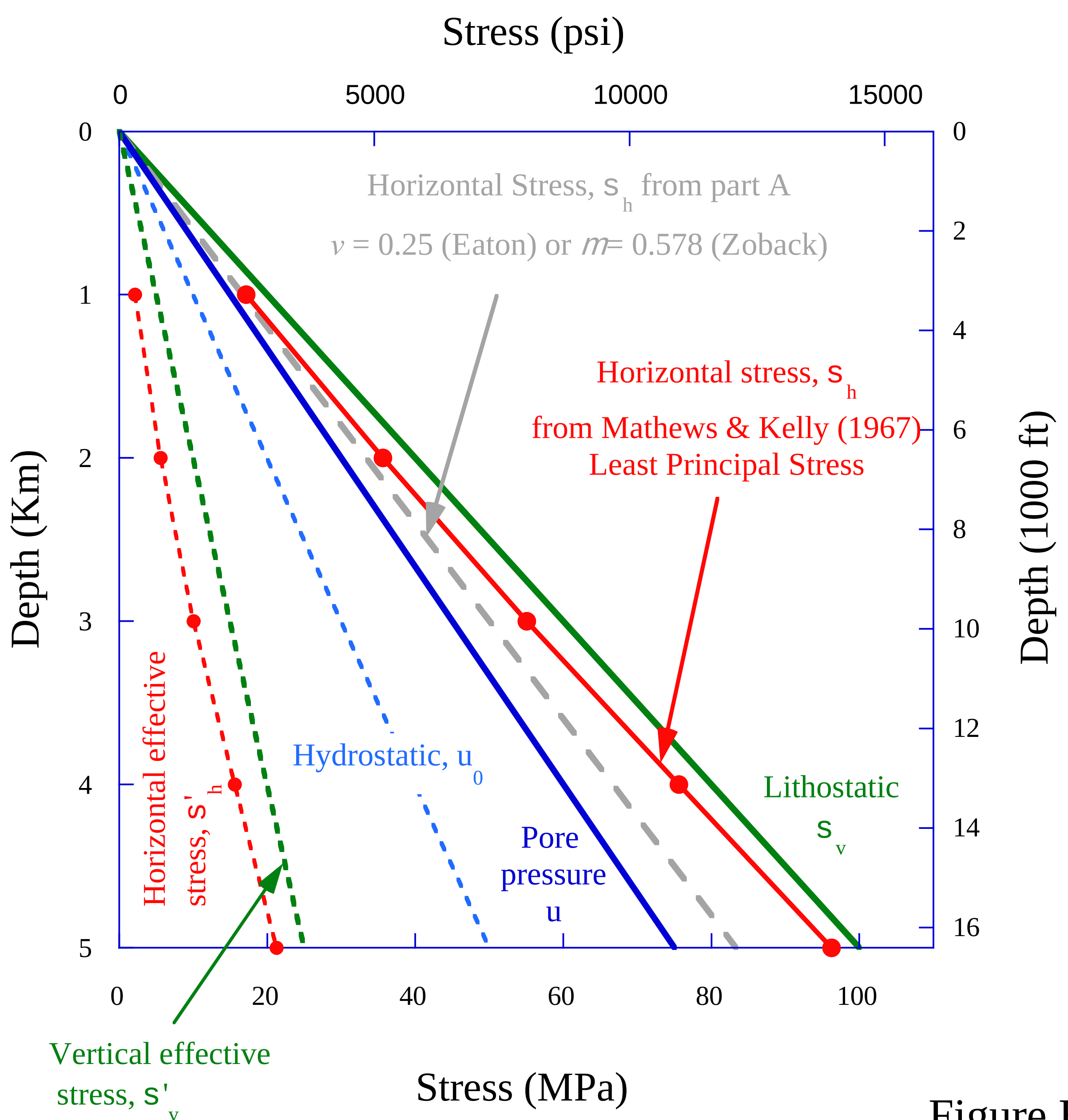
Stress (psi)
0
5000
10000
15000
Horizontal Stress, s_h from part A
v = 0.25 (Eaton) or m = 0.578 (Zoback)
Horizontal stress, s_h
from Mathews & Kelly (1967)
Least Principal Stress
Depth (Km)
Depth (1000 ft)
Horizontal effective stress, s'_h
Hydrostatic, u_0
Pore pressure u
Lithostatic s_v
Vertical effective stress, s'_v
Stress (MPa)
0
20
40
60
80
100
0
1
2
3
4
5
0
2
4
6
8
10
12
14
16

Figure B

C) APPLICATION: STRESSES AT A THRUST BELT

At a thrust belt, the horizontal stresses are elevated due to lateral shortening and hence the minimum principal stress is now vertical. In other words, $\sigma_3 = \sigma_v$ and $\sigma_1 = \sigma_h$.

Equation A2 then becomes:

$$\sigma_v = \left[\frac{1-\sin\phi}{1+\sin\phi}\right](\sigma_h - u) + u \qquad \text{Equation A.2}$$

and using ϕ= 30, $(\sigma_v - u) = [0.333](\sigma_h - u)$

or, $\sigma_h = [3](\sigma_v - u) + u$ Equation C1

From equation C1 and the vertical-stress and pore-pressure profiles calculated initially:

a. Horizontal total stress

$$(\sigma_h - u) = [3](\sigma_v - u)$$

Depth z = 0: σ_h = 0; z = 5km: σ_h = (3)x(100-75)+75 = 150 MPa

b. Horizontal effective stress

$$\sigma'_h = [3]\sigma'_v$$

Depth z = 0: σ'_h = 0; z = 5km: σ'_h = (3)x25= 75 MPa

Note that $\sigma_h = \sigma'_h + u$

These stress profiles are plotted in Figure C.

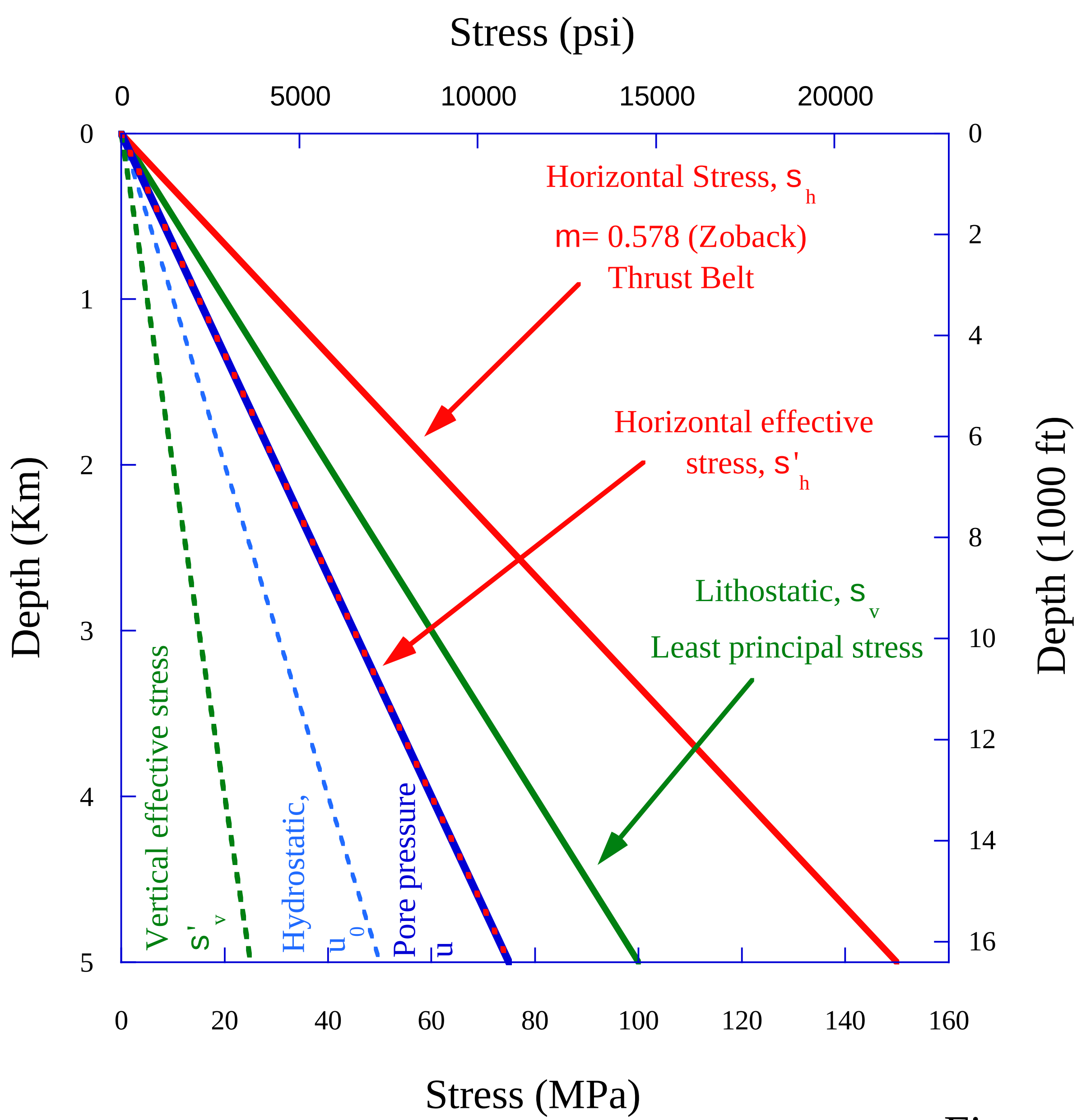
Stress (psi)
0
5000
10000
15000
20000
0
1
2
3
4
5
Depth (Km)
0
2
4
6
8
10
12
14
16
Depth (1000 ft)
Horizontal Stress, s h
m= 0.578 (Zoback)
Thrust Belt
Horizontal effective
stress, s' h
Lithostatic, s v
Least principal stress
Vertical effective stress
s' v
Hydrostatic,
u 0
Pore pressure
u
0
20
40
60
80
100
120
140
160
Stress (MPa)

Figure C

EXPERT REPORT

IN RE: OIL SPILL BY THE OIL RIG "DEEPWATER HORIZON" IN THE GULF OF MEXICO, ON APRIL 20, 2010

UNITED STATES DISTRICT COURT
EASTERN DISTRICT OF LOUISIANA
MDL NO. 2179, SECTION J
JUDGE BARBIER; MAGISTRATE SHUSHAN

OCTOBER 17, 2011

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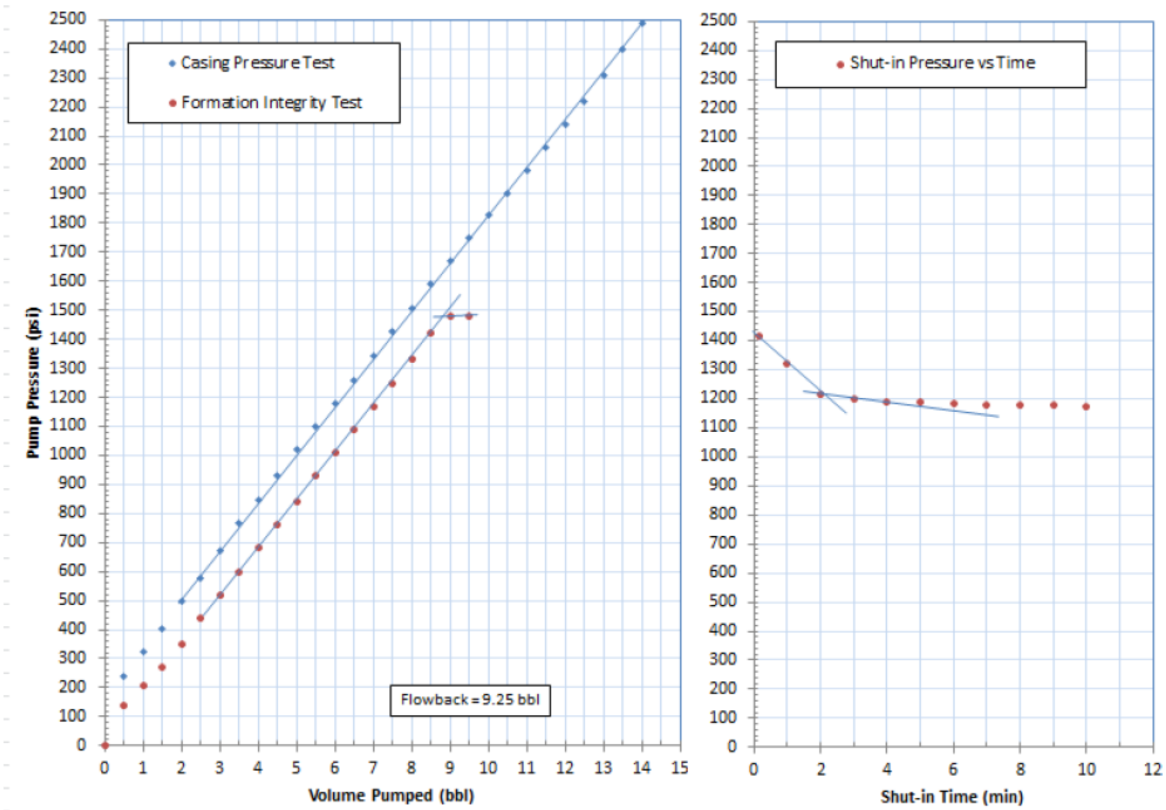

Figure 13: Pressure Integrity Test at 13-5/8 inch Liner Shoe

The results of my analysis are shown graphically in Figure 13. Basing the fracture gradient on the observed fracture initiation pressure of 1480 psi, the fracture gradient is calculated to be equivalent to a surface mud weight of 14.7 pounds per gallon. On this test, the fracture initiation pressure was the breakdown pressure. This is consistent with the 10 feet of formation being exposed consisting of an impermeable smooth borehole with no pre-existing defects or cracks and significant tensile strength and stress concentration near the borehole wall.

The fracture pressure at 13,145 feet calculated using the equivalent mud weight on bottom as measured by the Pressure While Drilling (PWD) tool was 10,175 psi. The overburden stress at a measured depth of 13,155 feet (true vertical depth of 13,145 feet), due to the weight of the sediments above, was calculated from well logs to be 9,632 psi. Thus the Leak-off test gave a value for fracture pressure that was about 543 psi higher than the calculated overburden stress. This is a reasonable value for the tensile strength for impermeable rock that was not previously fractured.

The shut-in pressure versus time plot is indicative of a fracture that closed slowly over a two minute period after pumping was stopped. The fracture closure pressure appears to be about 1,200 psi, which is equivalent to a surface mud weight equivalent to 14.3 pounds per gallon. However, fracture closure pressure generally cannot be accurately measured in mud because the mud starts to gel as soon as pumping is stopped.