HW-3d: Calculating Least Principal stresses

# ANSWERS

# General input:

Basin: depth 5 Km

Pore water: hydrostatic gradient: 10 MPa/km

 Overpressured gradient: 15 MPa/km

Sediments: Overburden gradient: 20 MPa/km

 Poisson’s ratio v’=0.25

***Calculation of overburden and pore pressure profiles***

The total vertical stress is calculated by intergrading the overburden: v = gz

Depth z = 0: v = 0; z = 5km: v = 100MPa

The hydrostatic pore pressure profile would be: u0 = wgz

 Depth z = 0: u0 = 0; z = 5km: u0 = 50MPa

However, the basin is overpressured with a gradient of 1.5gr/cc, therefore: u = gz

 Depth z = 0: u = 0; z = 5km: u = 75MPa

Finally, the effective vertical stress can be calculated by subtracting the pore pressures from the overburden: ’v = v - u

 Depth z = 0: ’v = 0; z = 5km: ’v = 25MPa

# A) Constant ratio between horizontal and vertical effective stresses

1. Eaton (1969) calculated the ratio of horizontal to vertical stresses using concepts of the elastic theory:

 $σ\_{h}= \left[\frac{v}{1-v}\right](σ\_{v}-u)+u$ Equation A.1

where $σ'\_{h}$ is the effective horizontal stress, $σ'\_{v}$ the effective vertical stress and u the pore pressure.

Using *v* = 0.25 and the values for the vertical stress and pore pressure calculated above, the horizontal stress profiles are:

1. Horizontal total stress

 $σ\_{h}= \left[\frac{1}{3}\right](σ\_{v}-u)+u$

 Depth z = 0: h = 0; z = 5km: h = (1/3)x(100-75)+75 = 83.33 MPa

1. Horizontal effective stress

 $σ'\_{h}= \left[\frac{1}{3}\right]σ'\_{v}$

 Depth z = 0: ’h = 0; z = 5km: ’h = (1/3)x25= 8.33 MPa

Note that h = ’h + u

These stress profiles are plotted in Figure A. The minimum principal stress is horizontal.

2. Zoback & Healy (1984) proposed a ratio of horizontal to vertical effective stresses based on the assumption that stresses in the Earth cannot exceed the frictional strength of pre-existing faults:

$σ\_{3}=\left[\frac{1-sinϕ}{1+sinϕ}\right]\left(σ\_{1}-u\right)+u $ Equation A.2

Where $ϕ$ is the friction angle ($μ=\tan(\left(ϕ\right)))$, 1 the maximum principal stress and 3 the minimum principal stress.

Generally, following deposition, the horizontal stress is lower than the vertical stress, and therefore the minimum principal stress is horizontal. In other words, 1 = v and 3 = h.

Using $ϕ=30$and the values for the vertical stress and pore pressure calculated above, the horizontal stress profiles are:

1. Horizontal total stress

 $σ\_{h}= \left[0.333\right](σ\_{v}-u)+u$

 Depth z = 0: h = 0; z = 5km: h = (0.333)x(100-75)+75 = 83.33 MPa

1. Horizontal effective stress

 $σ'\_{h}= \left[0.333\right]σ'\_{v}$

 Depth z = 0: ’h = 0; z = 5km: ’h = (0.333)x25= 8.33 MPa

Note that h = ’h + u

These stress profiles are plotted in Figure A. The minimum principal stress is horizontal.

Note that both approaches calculate the same stress ratio. This is a result of the values we chose for Poisson’s ratio and the friction angle. However it illustrates that these methods are approximations and have historically been used to fit existing data; they do not reflect the true behavior of the sediments during burial (see part 2.A of the homework).



# B) Stress ratio varying with depth

Various authors have proposed empirical curves that relate the stress ratio with depth (e.g. Eaton (1969), Matthews & Kelly (1967)).

Table B provides values for the stress ratio *Ki* based on the empirical relation proposed by Matthews & Kelly (1967) from the Louisiana Gulf Coast.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Depth (km) | *Ki* | v (MPa) | u (MPa) | ’h (MPa) | h (MPa) |
| 1 | 0.43 | 20 | 15 | 2.15 | 17.15 |
| 2 | 0.56 | 40 | 30 | 5.6 | 35.6 |
| 3 | 0.67 | 60 | 45 | 10.05 | 55.05 |
| 4 | 0.78 | 80 | 60 | 15.6 | 75.6 |
| 5 | 0.85 | 100 | 75 | 21.25 | 96.25 |

*Table B: Stress ratio Ki with depth from Matthews & Kelly (1967)*

We calculate the vertical stress by v = gz (as in the initial section of the solution) and the pore pressure by u = gz

Then, $σ\_{h}= K\_{i}(σ\_{v}-u)+u$ Equation B.1

Figure B plots the calculated stress values with depth. The figure also plots the constant ratio solution for comparison. Note that the stress rises with depth and approaches the overburden. Up to the depth of 5 km, the least principal stress remains horizontal.



# C) Application: Stresses at a thrust belt

At a thrust belt, the horizontal stresses are elevated due to lateral shortening and hence the minimum principal stress is now vertical. In other words, 3 = v and 1 = h.

Equation A2 then becomes:

 $σ\_{v}=\left[\frac{1-sinϕ}{1+sinϕ}\right]\left(σ\_{h}-u\right)+u $ Equation A.2

and using **= 30, $(σ\_{v}-u)= \left[0.333\right](σ\_{h}-u)$

or, $σ\_{h}= \left[3\right](σ\_{v}-u)+u$ Equation C1

From equation C1 and the vertical-stress and pore-pressure profiles calculated initially:

1. Horizontal total stress

 $(σ\_{h}-u)= \left[3\right](σ\_{v}-u)$

 Depth z = 0: h = 0; z = 5km: h = (3)x(100-75)+75 = 150 MPa

1. Horizontal effective stress

 $σ'\_{h}= \left[3\right]σ'\_{v}$

 Depth z = 0: ’h = 0; z = 5km: ’h = (3)x25= 75 MPa

Note that h = ’h + u

These stress profiles are plotted in Figure C.







