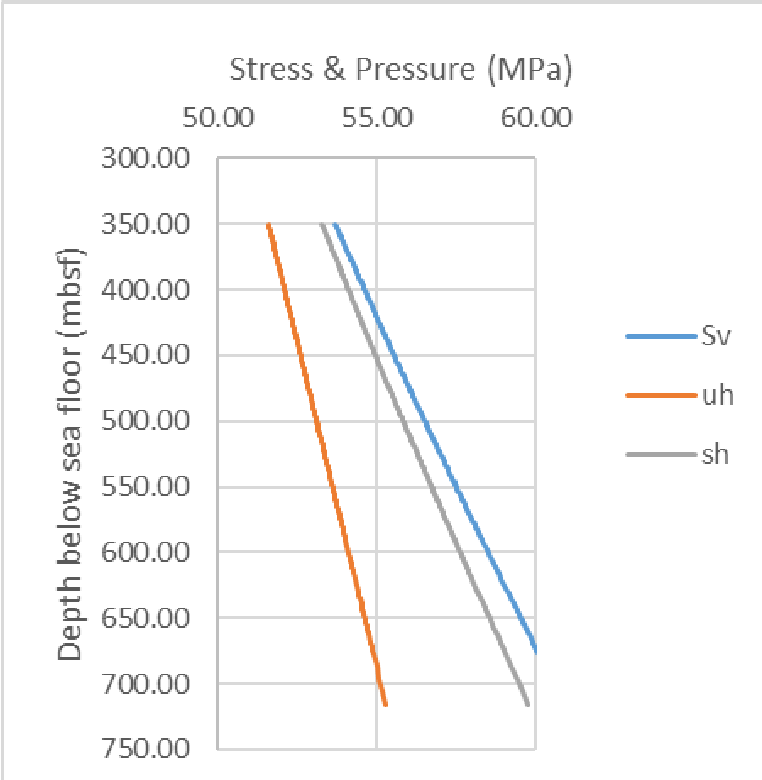
HW-3a: Compression Behavior in the Lower Shikoku of the Nankai Fold and Thrust Belt

ANSWERS

We will study the compression behavior in the Nankai Trough at IODP Site 1173. Figures 1 and 2 provide a general overview of this location.

1) See the spreadsheet associated with this homework. For Site 1173, please calculate the overburden from the seafloor downwards. Note that the data start at 350 meters below seafloor. I have provided the total vertical stress (σ_v) and the hydrostatic pressure at 350 mbsf (meters below seafloor) in the spread sheet. The water depth is 4795.7 meters. Assume that the pore pressure is hydrostatic.
    a. Construct a pressure-depth plot from the seafloor to total depth.
        i. Include hydrostatic pressure. Assume a water density of 1023 kg/m3.
        ii. Include the overburden stress. Calculate the overburden stress from 350 meters downward by integrating the bulk density, which you infer from the measured porosity (given in the spread sheet). Assume a solid sediment density of 2750 kg/m3 (this is slightly denser than quartz, but that is because clays are denser than quartz).
        iii. Include the least principal stress assuming uniaxial strain and that K_0 is 0.8 and that the pore pressure is hydrostatic.

In excel, I calculated density and void ratio. I integrated the density to get the overburden stress. I calculated the hydrostatic pressure. I used the stress ratio equation ($K_0 = \frac{\sigma_3'}{\sigma_1'}$) presented in class to determine the least principal stress (the horizontal total stress)



2) Two common expressions used to describe the compression behavior of mudrocks are the 'geotechnical' approach (Eq. 1) and the 'Hubbert' Approach (Eq.2):

$e_1 = e_o - C_c \log\left(\frac{\sigma_1'}{\sigma_o'}\right)$ Eq. 1

$n = n_0 e^{-\beta\sigma_v'}$ Eq. 2

$\sigma_v' = \sigma_v - u$ Eq. 3

$e = \frac{n}{1-n}$ Eq. 4

e_o is a reference void ratio commonly taken at a vertical effective stress, $\sigma_o{}'$, of unity, and Cc is a compression coefficient. In Eq. 2, n is porosity, n_o is a reference porosity at an effective stress of zero and β is a compressibility parameter. Equation 3 is the effective stress equation. In locations where the pore pressure is hydrostatic, u is the hydrostatic pressure u_h.

a. Plot void ratio (e) vs. the log of hydrostatic effective stress ($\sigma_{vh}' = \sigma_v - u_h$). Use regression to determine the parameters $e_0, and\ C_c$. After you have derived $e_0, and\ C_c$, then please substitute them in Eq. 1 and plot this line.
b. Plot porosity (n) vs. hydrostatic effective stress (σ_{vh}') and use regression to determine the parameters n_o, β. After you have derived n_o & β, then please substitute them in Eq. 2 and plot this line.
c. Please determine the value of m_v at 3.0 and 4.0 MPa. Remember that the coefficient of compressibility is the slope of the void ratio vs. effective stress plot divided by 1+void ratio at the location the slope is taken:

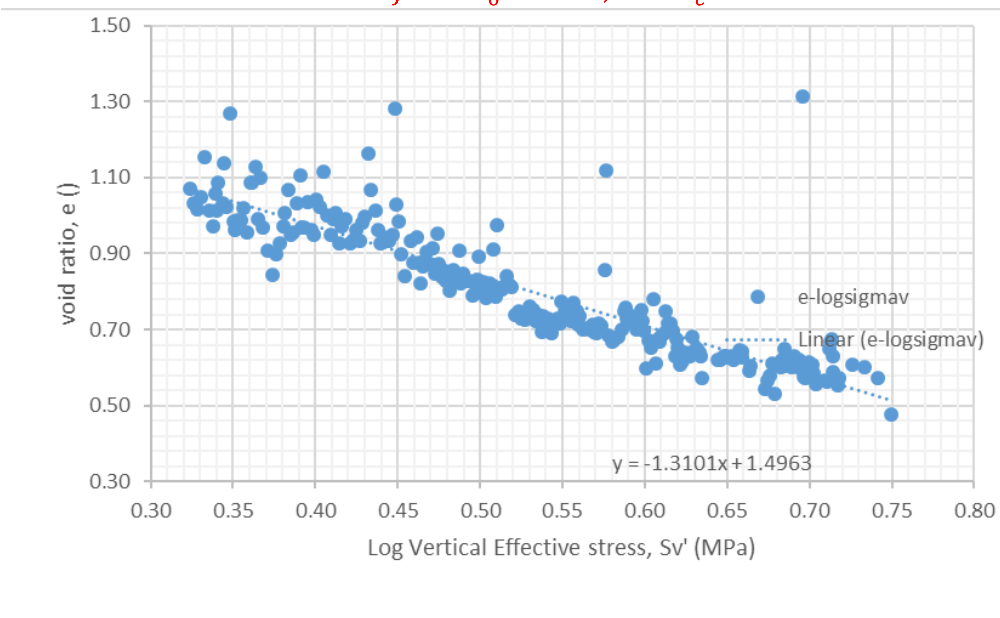
$$m_v = \left(\frac{-1}{1+e}\right)\frac{de}{d\sigma'} \qquad \text{Eq. 5}$$

d. Plot porosity vs. depth for the 1173 well adjacent to your pressure vs. depth plot (#1 above).

Step 1 is to determine the compression parameters for the 1173 well using the geotechnical approach. In the figure below, I have performed a linear regression of the following equation:

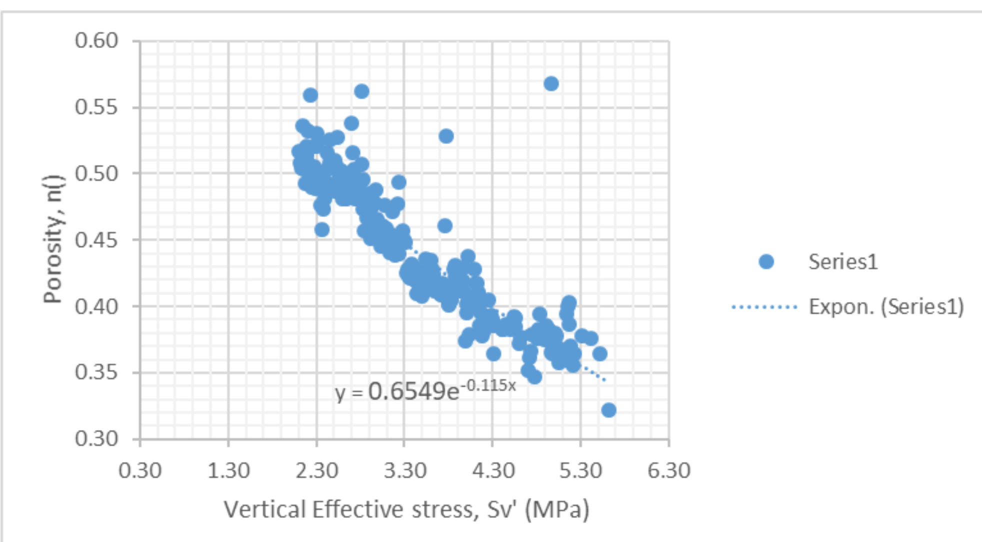
$$e = e_o + C_c \log(\sigma_v')$$

$$we\ find: e_0 = 1.50, and\ C_c = 1.31$$



I have repeated the exercise here for the Hubbert approach:

$$n = n_o e^{-\beta \sigma_v'}$$



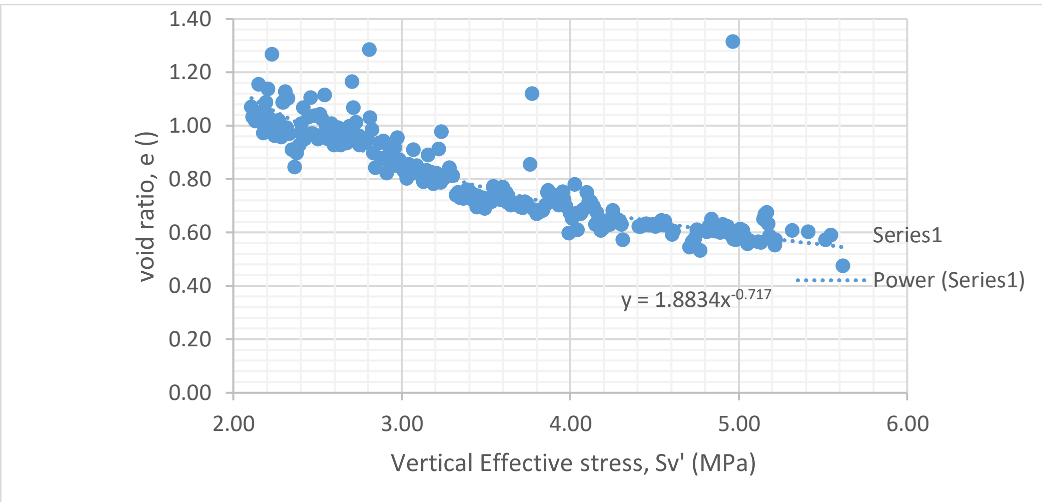
I find:

$$n_o = 0.65$$
$$\beta = -0.115\ MPa^{-1}$$

I leave it to you to plot the porosity vs. depth

m_v is defined in Eq. 5:

A plot of void ratio vs effective stress is:



I fit a power-law function to the plot of void ratio vs. effective stress. In the attached page, I calculate m_v for the stress at 3.0 and 4.0 MPa. I find mv = 0.11 MPa^{-1} at 3 MPa and 0.07 MPa^{-1} at 4 MPa. This demonstrates that m_v decreases with increasing stress. This implies the rock gets stiffer as it compacts. This makes sense because one can envision more particle contacts with decreasing porosity.

I have fit a power law to the plot of void ratio to effective stress.

I find:

$$e = 1.88(\sigma_v')^{-0.717} \quad (Eq. \#1)$$

I use Equation #1 to derive e at 3 and 4 mPa:

$e_{3mPa} = 0.85$

$e_{4mPa} = 0.69$

$$m_v = \frac{1}{1+e}\,\frac{-de}{d\sigma_v'} \quad (Eq \#2)$$

To determine $\frac{de}{d\sigma_v'}$, I take the derivative of Eqn #1:

$$e = a\sigma_v'^{\,b}$$

$$de = ab\sigma_v'^{\,(b-1)}\,d\sigma_v'$$

$$\frac{de}{d\sigma_v'} = ab\sigma_v'^{\,(b-1)} \quad (Eq. \#3)$$

Eq #3: @ 3 mPA $\frac{de}{d\sigma_v'} = 1.88\cdot(-.717)(3mpa)^{(-0.717-1)} = -0.204$

Eq #3 @ 4mPa " " " $(4mpa)$ " $= -.12$

Eq #2 m_v @ 3mPa $= \left[\frac{1}{1+.85}\right]\left[.204\right] = \boxed{0.11\ mPa^{-1}}$

Eq #2 m_v @ 4mPa $= \left[\frac{1}{1+.69}\right]\left[.12\right] = \boxed{0.07\ mPa^{-1}}$