Quantitative and mathematical concepts/ skills/ competencies

2^{nd} Summit on the Future of Undergraduate Geoscience Education

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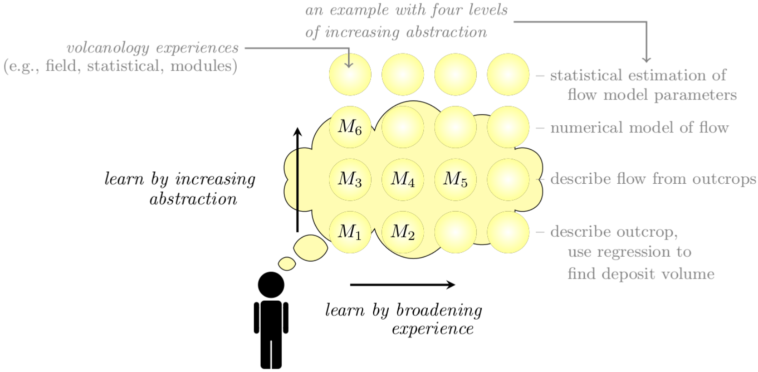
Objective

Increase quantitative literacy, mathematical competency and algorithmic thinking among geoscience students

A schema of schemas...



QL in Geology Courses

- To address issues in quantitative literacy generally, a structure and format for modules called spreadsheets across the curriculum (SSAC) was created by Len Vacher (USF) for a materials development project in geoscience education of the same name
- SSAC modules are highly formatted, stand-alone, computer-based activities, each designed to engage students in a problem-solving activity that invokes their interest. Every SSAC module is a PowerPoint presentation, usually consisting of 12–18 slides, with embedded Excel spreadsheets, and perhaps including supplementary slides with ancillary information.
- The core of a module consists of slides that lead the students to identify a quantitative problem, consider how to design a solution to the problem, and build their own spreadsheets to solve the problem.

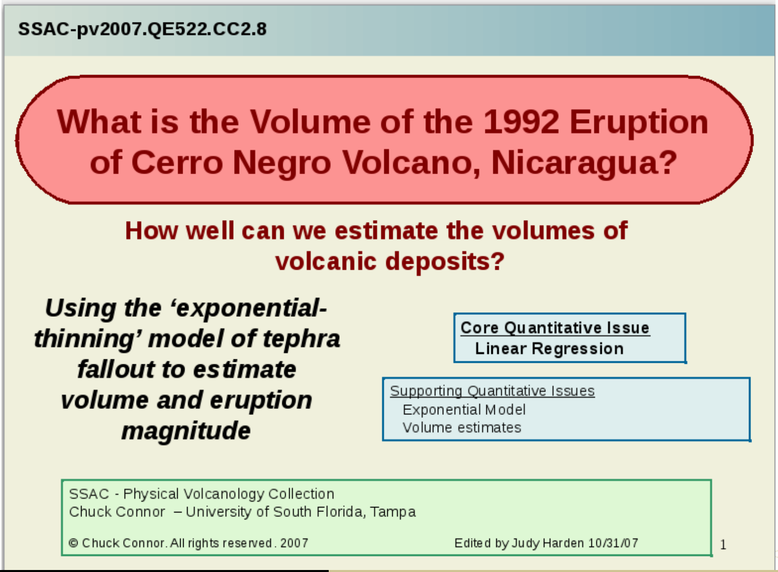
QL in Geology Courses

For the physical volcanology collection of SSAC modules, each problem is developed to parallel disciplinary course material. Each module has:

- A title slide that notes the quantitative concepts and skills introduced by the module
- One or more slides that present volcanological background
- One slide that clearly states a volcanological problem
- A set of slides dedicated to designing a plan for solving the problem
- A set of slides for carrying out the solution to the problem (using Excel)
- A slide that allows students to reflect on the solution to the problem and that provides additional reference material
- An end-of-module-assignment slide

All are freely available on the SERC website
(Google: SERC physical volcanology collection)

Physical Volcanology modules



Preview

This module presents a calculation of the volume of the 1992 eruption of Cerro Negro volcano, Nicaragua.

Slide 3-6 *give some background on tephra fallout and the 1992 eruption.*

Slide 7 *states the problem. What is the volume of the tephra fallout deposit?*

Slides 8 and 9 *develop a plan for solving the problem. An exponential thinning model is assumed to describe the change in thickness of the deposit with distance from the volcano. This model is used to estimate the volume of the deposit.*

Slides 10-13 *illustrate the solution of the problem, developing a spreadsheet to calculate the volume.*

Slide 14 *discusses the point of the module and provides a broader volcanological context.*

Slide 15 *consists of some questions that constitute your homework assignment.*

Slides 16-20 *are endnotes for elaboration and reference.*

Slides 21 and 22 *contain additional information required to answer question 4 in the end of module assignments.*

Physical Volcanology modules

Background

What is tephra fallout?

tephra – a term that refers to all particles ejected explosively from a volcano, regardless of size, shape, or composition.

The term was introduced by the famous Icelandic volcanologist Thorarinsson to describe particles that travel through the air, are carried by wind, and eventually land on the ground.

Therefore, tephra fallout deposits (varieties called "airfall deposits", "pyroclastic fallout deposits") are completely comprised of tephra and have characteristics that distinguish them from other types of flow phenomena. Tephra fallout deposits mantle the terrain and thin, often exponentially, with distance from the volcano.

Photo by C. Connor

Under the plume of the 1992 eruption of Cerro Negro volcano, Nicaragua, during the waning stages of the eruption. More than 80 cm of tephra had already accumulated in this area when the photograph was taken, virtually destroying the forest of rich tropical growth that covered the area just a few days before.

For more about tephra terminology:

http://volcanoes.usgs.gov/Hazards/What/Tephra/tephraterms.html

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Background

Cerro Negro volcano is a cinder cone located in the Central American volcanic arc, created by the subduction of the Cocos plate beneath the Caribbean plate. Cerro Negro first formed in 1850 and has experienced more than 20 eruptions since its formation. One of the largest of these eruptions occurred in April, 1992. With little warning, a sub-Plinian eruption column developed that reached about 7 km height. When the 2 day eruption was over, 20,000 people had been evacuated, mostly due to failed water systems. Crop damage was widespread and catastrophic, and some roofs had collapsed in the nearby city of Leon, the second largest city in Nicaragua.

The 1992 Eruption of Cerro Negro

Photo by C. Connor

Building collapse as a result of tephra fallout during the 1992 eruption of Cerro Negro. In this area, just east of the city of Leon, tephra fallout accumulation was only about 4 cm, but sufficient to cause collapse of this poorly constructed roof.

Location of Cerro Negro volcano

Learn more about the consequences of the eruption

Quantitative and mathematical concepts/ skills/ competencies

Objectives

Math in Geoscience Courses

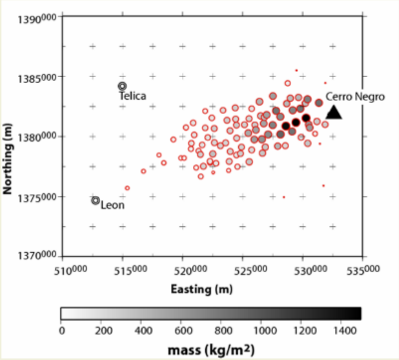
Geocomp. Course

Problem

What is the volume of the 1992 tephra fallout deposit?

Data are required to estimate the volume of the tephra fallout deposit. In this case, the data comes from pits dug in the tephra fallout deposit. These pits are used to record thickness of the deposit, density of the deposit, and grain size. On the map, tephra fallout is represented as mass per unit area.

Given a deposit thickness and deposit density – make sure you can calculate the mass per unit area – a value much more useful than thickness for understanding potential damage.



Connor and Connor, 2006

Learn more about how this map was made

Physical Volcanology modules

Designing a Plan, Part 1

Given information about the area enclosed by specific isopach lines, estimate the volume of the 1992 eruption.

You will need to:

- **Plot the natural logarithm of isopach thickness (m) against the square root of isopach area (km).**
- **Find a best-fit linear model for these data.**
- **Estimate parameters (slope and y-intercept) from your best-fit linear model.**
- **Estimate deposit volume from these parameters.**

Give answer in cubic kilometers.

Notes:

The model you will use is based on the exponential-thinning model. That is, it is assumed that the thickness of the deposit, on average, decays exponentially away from the vent of the volcano (hence ln(Thickness) is plotted).

Square-root of isopach area is used in place of distance from the volcano to average out the effects of the wind.

You will use the excel “trend line” function to fit a line to the data and to estimate the parameters of the model.

With these parameters in hand, you will estimate the volume of the deposit.

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Designing a Plan, Part 2

Use the exponential-thinning model of Pyle (1989) to estimate the volume.

In the exponential-thinning model, deposit thickness, *T*, depends on area of the deposit, *A*, that has thickness greater than *T*, an initial thickness at the vent, T_o, and a thinning constant, *k*:

$$T = T_o \exp\left[-kA^{1/2}\right]$$

The equivalent half-thickness is the average distance from the vent at which the deposit reaches one half its initial thickness. This is a measure of dispersion of the deposit. Think of b_T as equivalent to the half-life in radioactive decay.

$$b_T = \ln(2) / k\sqrt{\pi}$$

The volume of the deposit is then estimated using the parameters T_o and b_T:

$$V = 13.08 T_o b_T^2$$

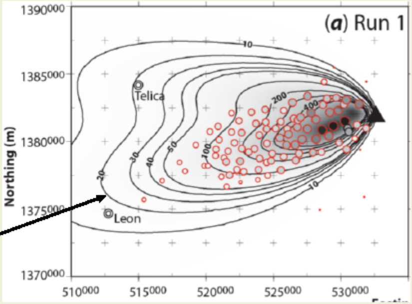
Learn about where these equations come from

Carrying out the Plan, Part 1: Isopach area

The first step is to convert information on the isopach (or isomass) map to a form used in the equations to calculate volume. In this case, you are given the area enclosed by isopachs of a given thickness. For example, the 2 cm isopach encloses approximately 181 km² area.

Volume is determined by estimating the parameters T_o and b_T using the equations of the previous slide.

	B	C	D
2	***Calculate the Volume of the Tephra***		
3			
4	***Given***		
5	Isopach Area	Thickness	
6	(km^2)	(m)	
7	16.25	0.4	
8	30.63	0.2	
9	58.87	0.1	
10	95.75	0.05	



Isomass map (kg m^{-2}) for the 1992 eruption
From Connor and Connor, 2006.

20 kg m^{-2} is equivalent to 2 cm thickness, if the density of the tephra fallout deposit is 1000 kg m^{-3}. Please be sure you understand this conversion.

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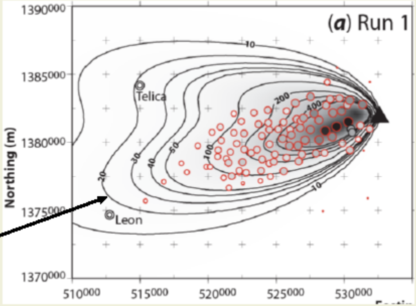

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Carrying Out the Plan, 4: Spreadsheet to Calculate the Volume

Once k and $\ln(T_o)$ are estimated, the values of b_T and volume are determined:

$$b_T = \frac{\ln(2)}{k\sqrt{\pi}}$$

$$V = 13.08 T_o b_T^2$$

	B	C	D	
2	***Calculate the Volume of the Tephra Deposit***			
3				
4	***Given***		***Calculated***	
5	Isopach Area	Thickness	Sqrt (Area)	Ln (Thi
6	(km^2)	(m)	(km)	Ln
7	16.25	0.4	4.031129	-0.9
8	30.63	0.2	5.534438	-1.6
9	58.87	0.1	7.672679	-2.3
10	95.75	0.05	9.785193	-2.9
11	181.56	0.02	13.474420	-3.9
12	275.10	0.01	16.586139	-4.6
13				
14	k	0.289		
15	$\ln(T_o)$	0.022		
16	b_T	1.353171	km	

These parameters are estimated by linear regression, using the "trend line" function in Excel.

Modify your spreadsheet to perform the final tasks in your volume calculation.

Be sure you keep careful track of units in this calculation.

Be sure you can easily change the units of volume from km^3 to m^3.

Verify that the 1992 eruption of Cerro Negro was VEI 3.

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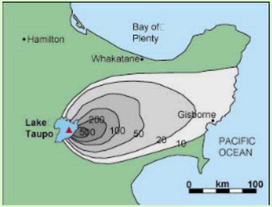
End of Module Assignments

1. Make sure you turn in a spreadsheet showing your estimated volume of the 1992 Cerro Negro eruption.
2. Given your volume estimate, assume the density of the tephra deposit is 1000 kg m^{-3}, and the density of the melt that actually erupted was 2800 kg m^{-3}. What was the volume of melt (as opposed to tephra) erupted? This latter volume is often referred to as the dense rock equivalent (DRE) volume.
3. Given the following table of information, use your spreadsheet to make volume estimates for the 1902 eruption of Santa Maria (Guatemala) and the AD 186 Taupo eruption (New Zealand). Estimate the VEI of these eruptions.

Santa Maria		Taupo	
Area (km^2)	Thickness (cm)	Area (km^2)	Thickness (cm)
255	150	441	138
1483	100	1369	97
5932	50	3025	66
27699	10	8100	30
86642	1	14400	15



Taupo isopach (Wilson and Walker 1985 from *http://volcano.und.edu/vwdocs/volc_images/australia/new_zealand/taupo.html*)

4. Given the information about the 2450 BP eruption of Pululahua volcano (Ecuador) on slides 21 and 22: (a) contour the thickness values to create an isopach map, (b) assuming circular isopachs, calculate the area enclosed by a set of at least 5 isopachs, (c) estimate the volume of the deposit (Please thank Alain Volentik for data used in this problem!)

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Physical Volcanology modules

So far, these 9 modules have been used in 11 physical volcanology sections at USF and, in some form, at 15+ other universities in North America and Europe.

Lessons learned:

- Modules help students avoid "short cuts" in problem solving
- Most students find the modules to be highly challenging (time consuming)
- Student performance improves when they encounter the same module format in multiple courses (geophysics, hydrogeology)
- Qualitatively, alums report "learning excel" and "learning to solve problems" as most important course outcome
- Modules are time-consuming for faculty to develop (shared model needed, e.g., SERC)

A dedicated geocomputation course at the sophomore / junior level

Course objective: To enhance computational skills and increase quantitative literacy in the context of geology. To increase comfort with geological-mathematical problem solving and communication about quantitative material.

- Now a required course at USF
- Little new math is introduced – mostly places math in geoscience context
- Dedicated to problem solving
- Pathway to scientific computing and related problem-solving skills required in grad school and industry.

A dedicated geocomputation course at the sophomore / junior level

Course topics:

- Introduction to quantitative literacy and problem solving
- Numbers and number sense – finding your way around a map
- Sets, Venn diagrams, and graph theory – learning about data sets, databases, and coding an outcrop
- Quantities, units, and dimensional analysis, significant figures and error
- Proportion and percentages, sums, averages and integrals
- Logs and log scales
- Circles and angles, distance and direction, vectors
- Probability
- Algorithmic thinking

Problem solving example

A laser rangefinder is an instrument that sends out a pulse of light and senses time required for the light to reach an object, like an outcrop, reflect off the object and return to the laser. Suppose the flight time for one measurement is 210 nanoseconds. Light travels 0.299792458 meters in one nanosecond.

How far does the light pulse travel?

Often the speed of light is approximated as 1 foot per nanosecond. How many feet does the light pulse travel using this approximation and what is the percentage error compared to the more accurate speed given in meters per second?

Problem solving example

Lots of natural processes involve diffusion. Diffusion can be expressed in one-dimension as:

$$\frac{\partial C}{\partial t} = \alpha \frac{\partial^2 C}{\partial x^2}$$

This is a partial differential equation, in which C is concentration (such as the concentration of a pollutant in groundwater, or the concentration of sediment in a stream), t is time and x is distance. The ∂ symbol refers to "change in", like the symbol Δ, and so has no units. In words, the equation says that the change in concentration with respect to time is equal to a proportionality constant, α, times the rate of change in concentration with respect to distance, x. Although partial differential equations can be complicated to solve, it is easy to check their dimensionality. In checking the dimensionality, just ignore the ∂, or "change in" symbol. Taking it one step at a time

$$\frac{\partial C}{\partial t} = \frac{kg}{m^3}\frac{1}{s} = \frac{M}{L^3 T}$$

and

$$\frac{\partial^2 C}{\partial x^2} = \frac{kg}{m^3}\frac{1}{m^2} = \frac{M}{L^5}$$

What are the SI and fundamental units of the proportionality constant α, which is called the diffusivity, and also called the diffusion coefficient?

A dedicated geocomputation course at the sophomore / junior level

Some lessons learned:

- Students are challenged by problem-solving in geology context
  - *necessary to revisit lots of basic math in geology context*
  - *students need to solve lots of problems*
- Word problem format and presentation limits cognitive load
  - *most problems worked with pencil and paper / some Excel*
  - *start problems as a group*
  - *discuss solutions as a group*
- Word problem development is time-consuming for faculty
  - *create an open-access model (Wiki)*
  - *challenge students to create their own geology word problems*