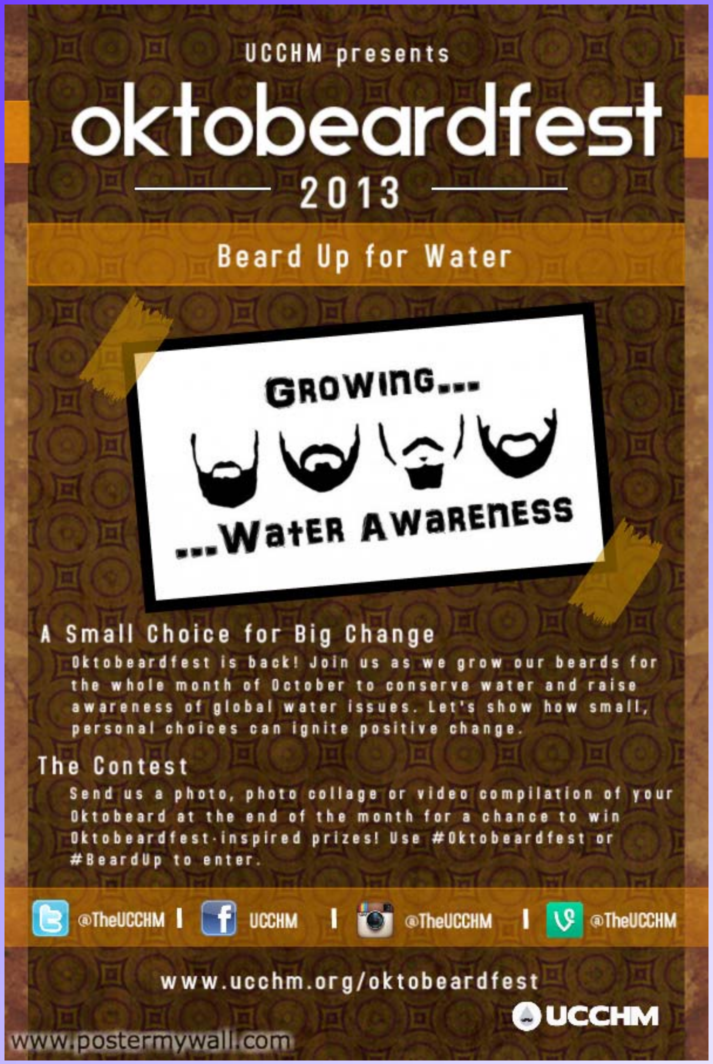
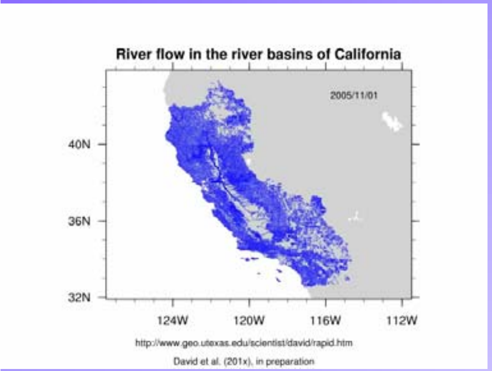
# Growing water awareness





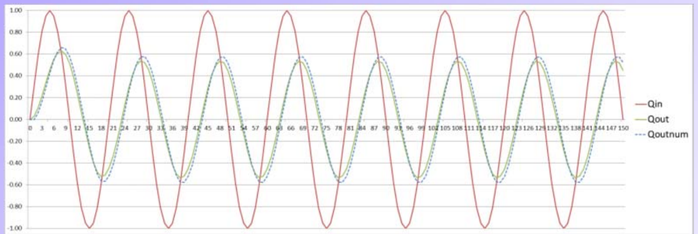


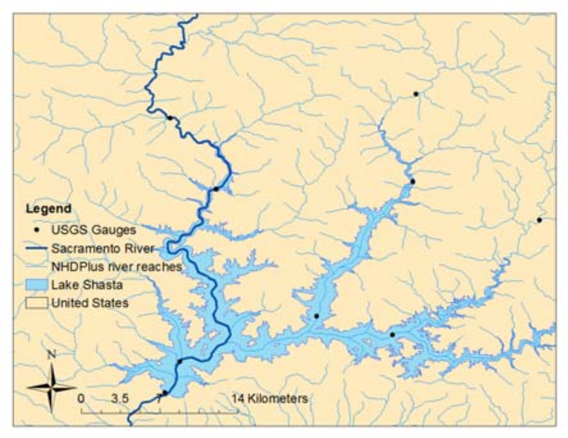

# Inclusion of a simple reservoir model in regional-scale surface water modeling



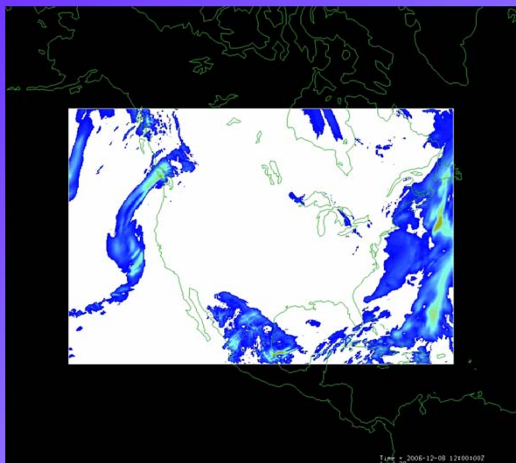

Cédric H. David
James S. Famiglietti

Texas Water Forum
15 Oct 2013

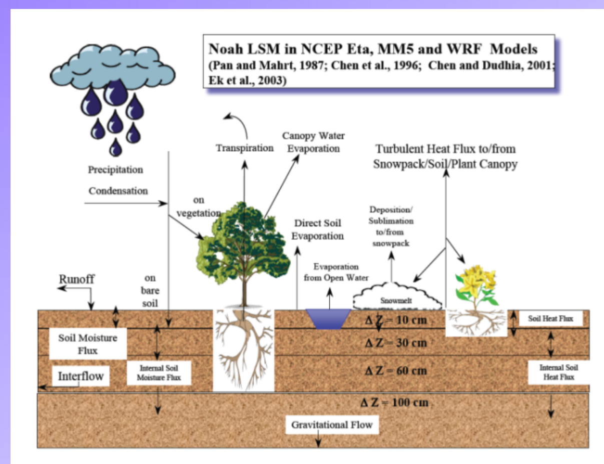


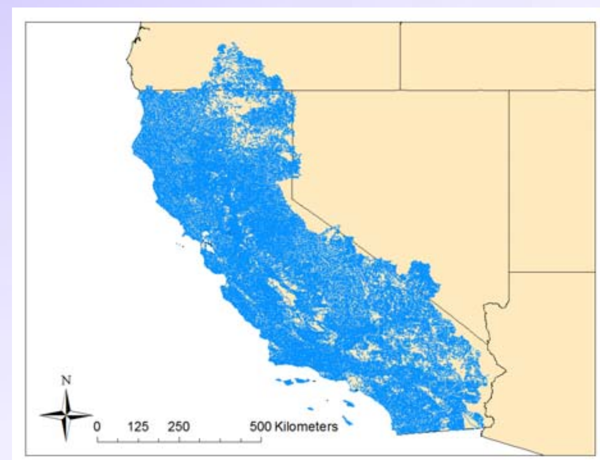
# Integrated River Modeling



Atmospheric Model or Dataset



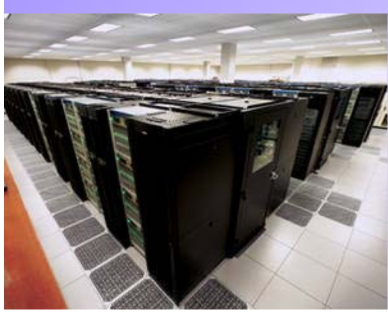

Land Surface Model

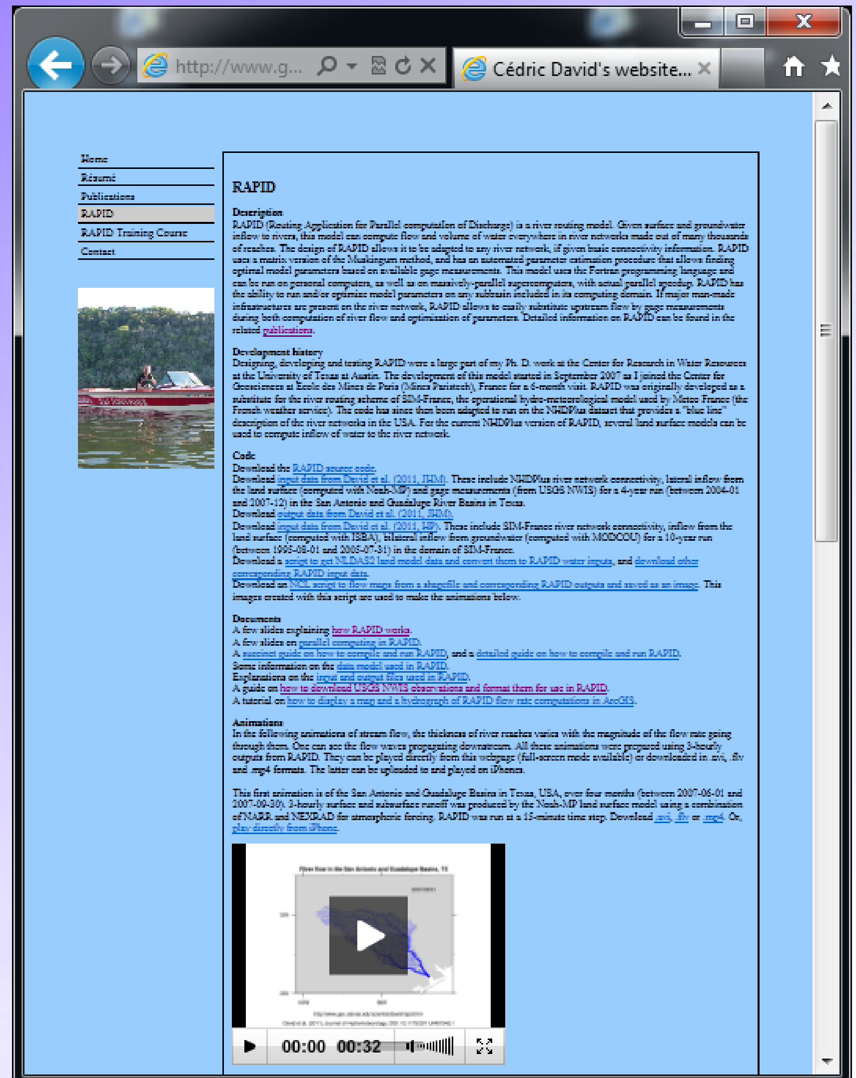



“Blue Line” River Network - High-Performance Computing River Network Model

# RAPID

- **R**outing **A**pplication for **P**arallel computat**I**on of **D**ischarge)
- Computes flow and optimizes model parameters
- Model code, input data and animations are available online
- Can run on supercomputers






http://www.ucchm.org/david/rapid.htm

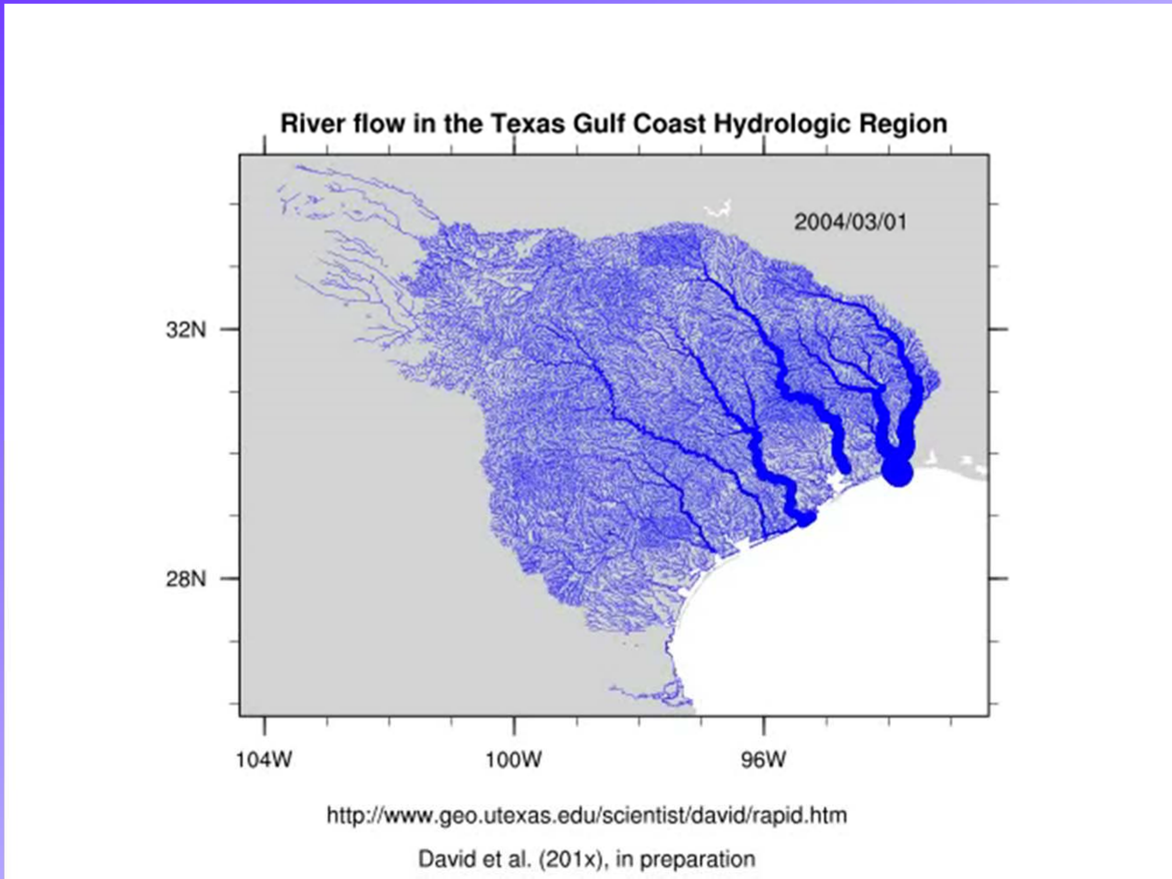
# Who made RAPID possible?

- UCAR Advance Study Program
- CUAHSI Hydrologic Information System (NSF EAR-0413265)
- NASA Interdisciplinary Science Projects (NNX07AL79G and NNX11AE42G)
- Ecole des Mines de Paris
- AGU Horton (Hydrology) Research Grant
- David Maidment, Florence Habets, Zong-Liang Yang, David Gochis, Jay Famiglietti





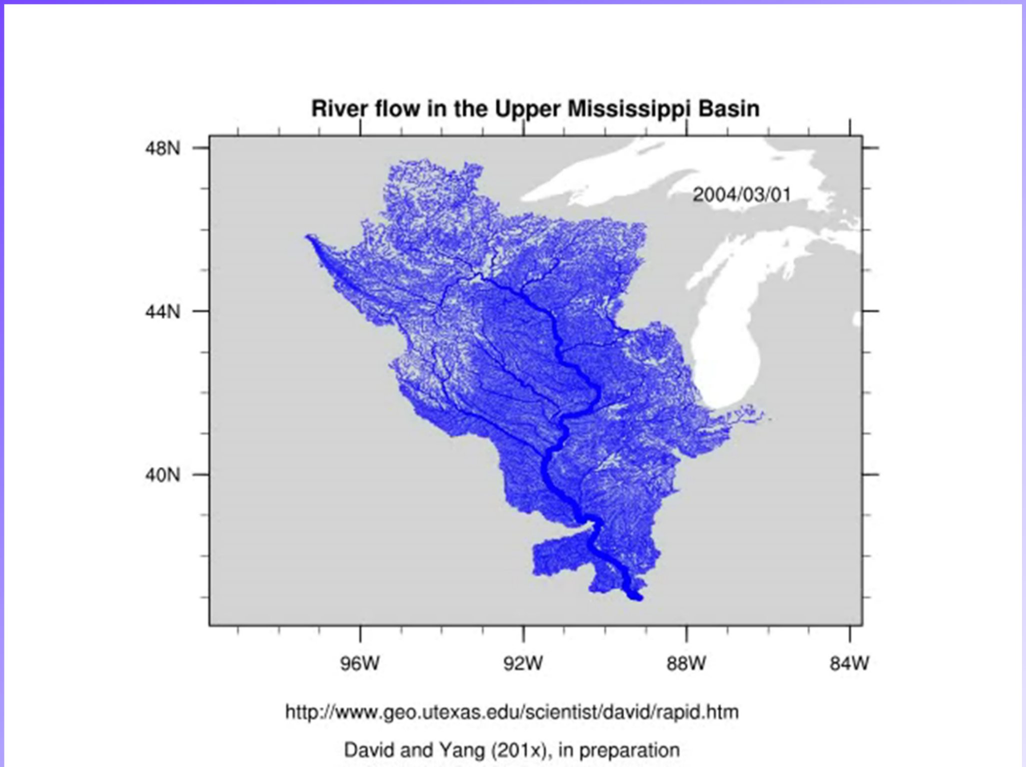
# Last we talked…



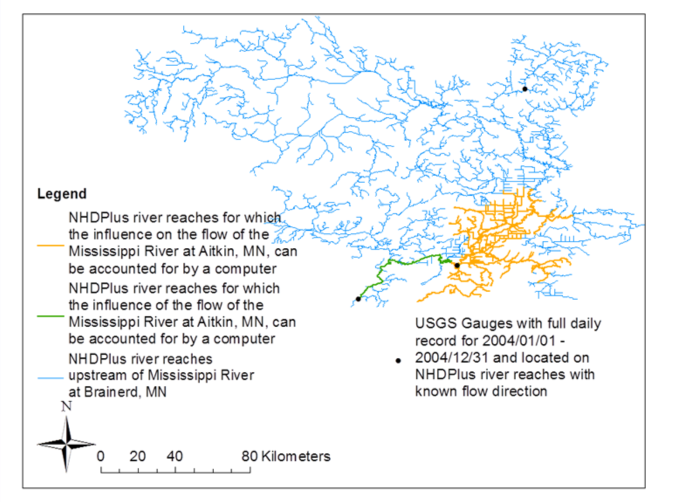

David et al. (2013a Env. Model. & Soft)

Water Forum 1
13 Feb 2012

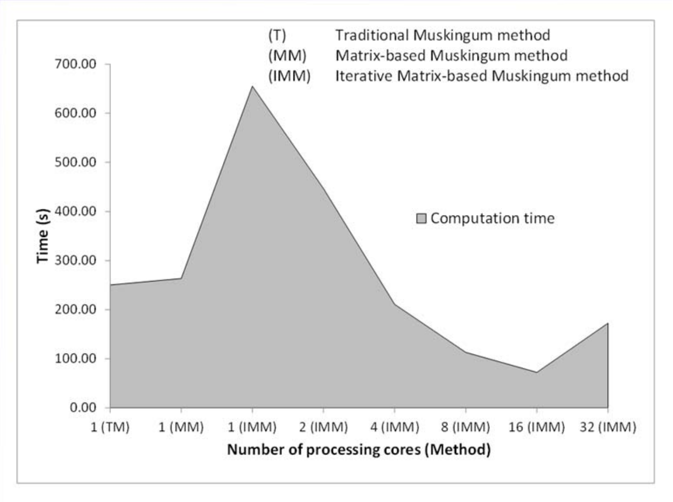
# Since then…



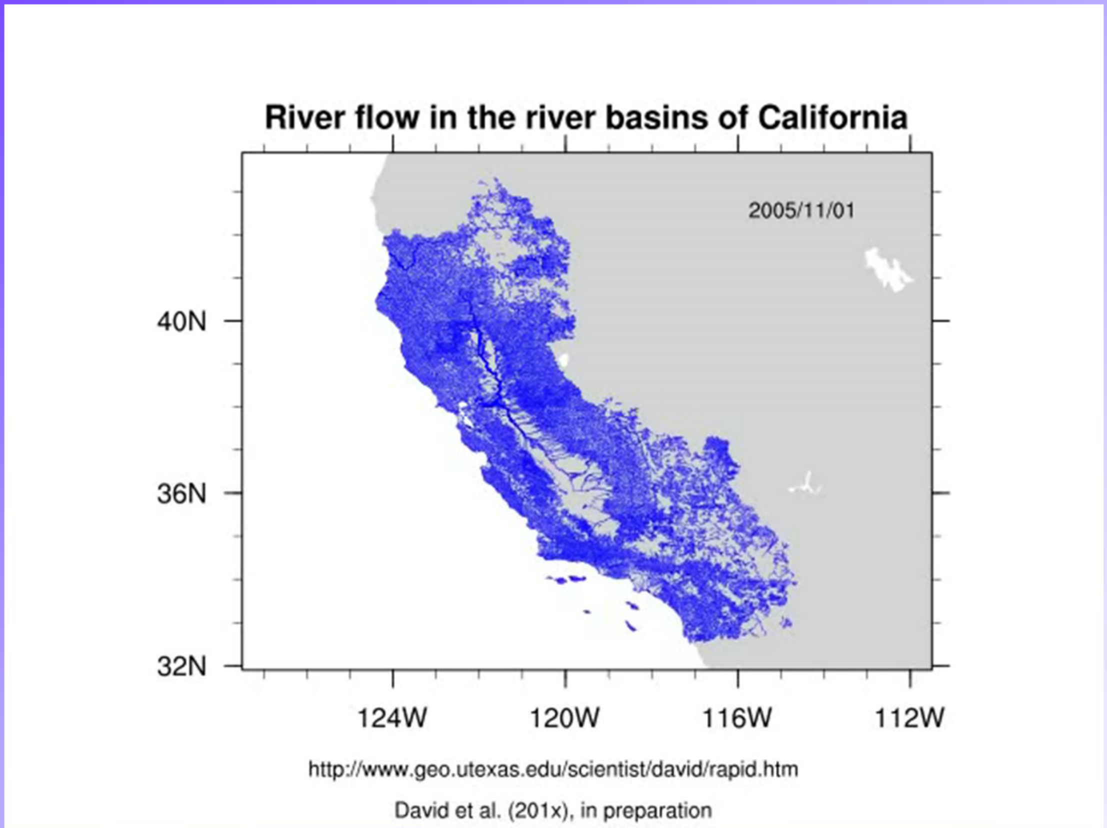

David et al. (2013b Water Resour. Res.)



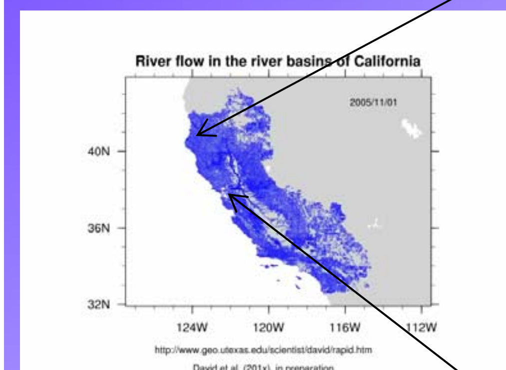

Flow wave propagation and parallel computing

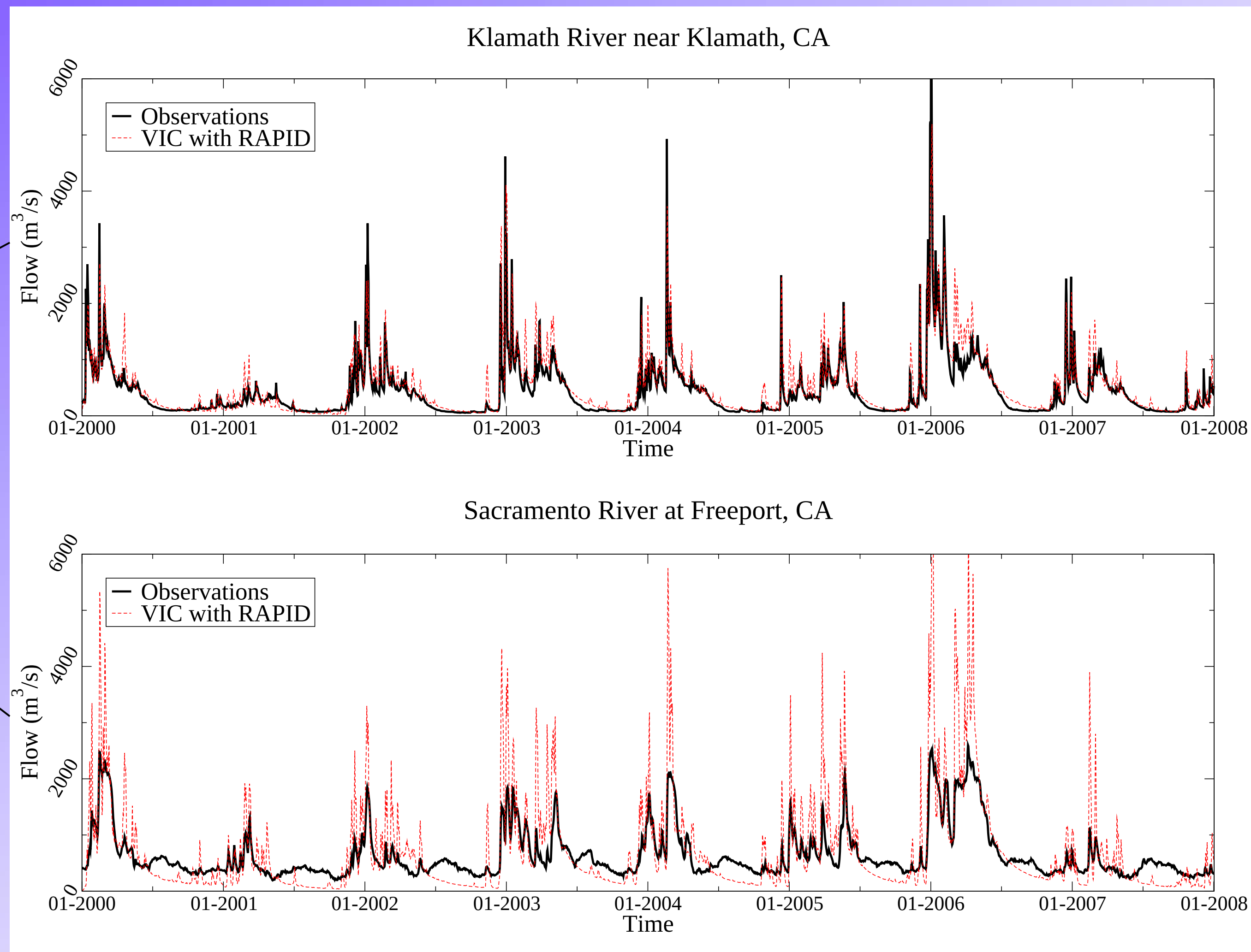


# Also, moved to California

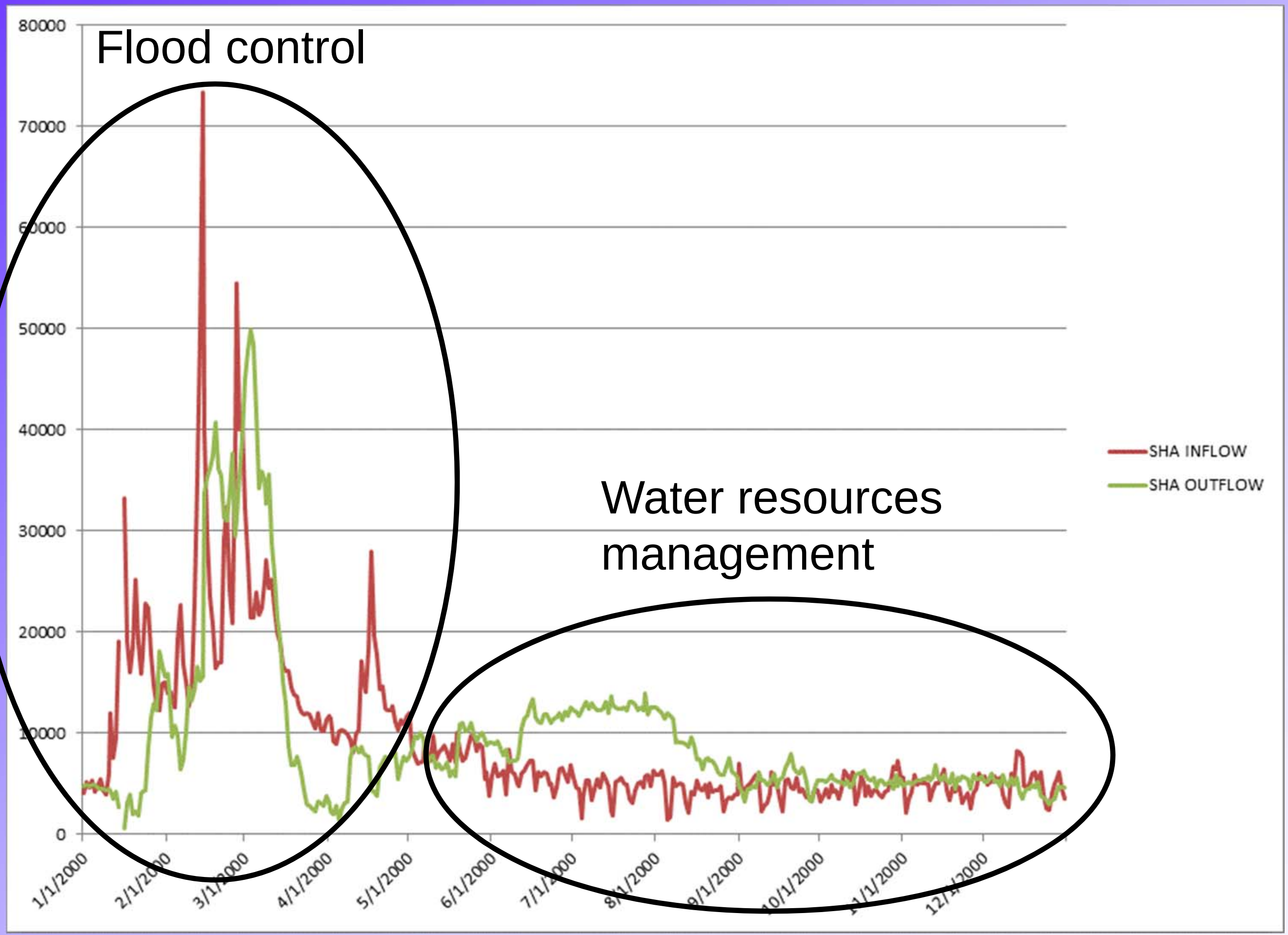


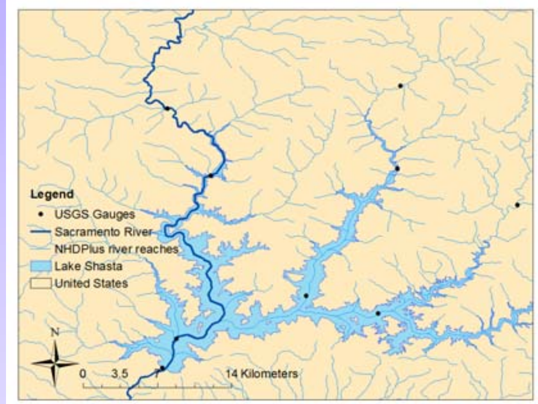

# Reservoirs and river modeling



# What does a reservoir do?

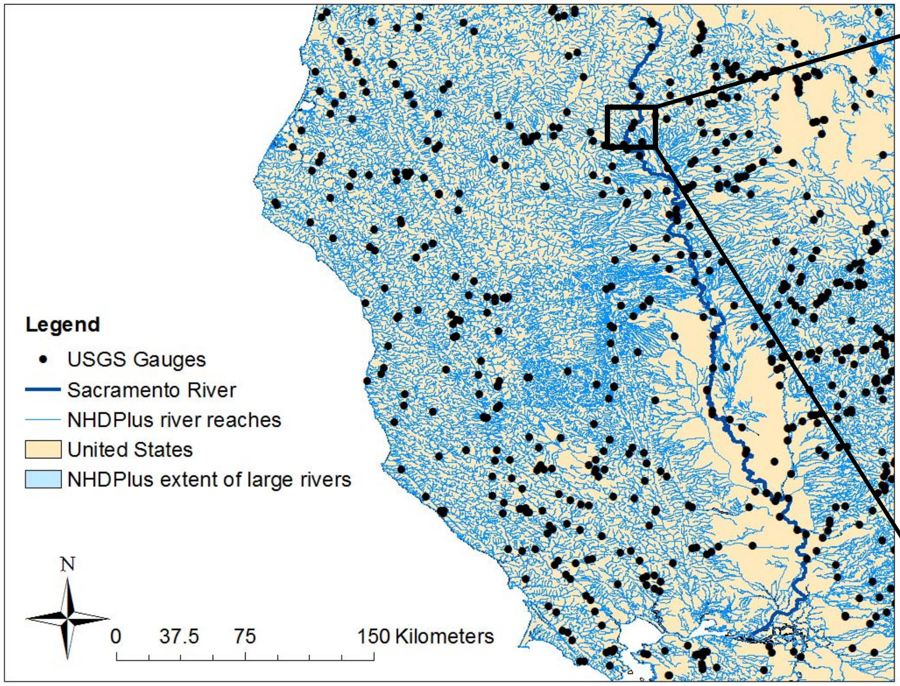


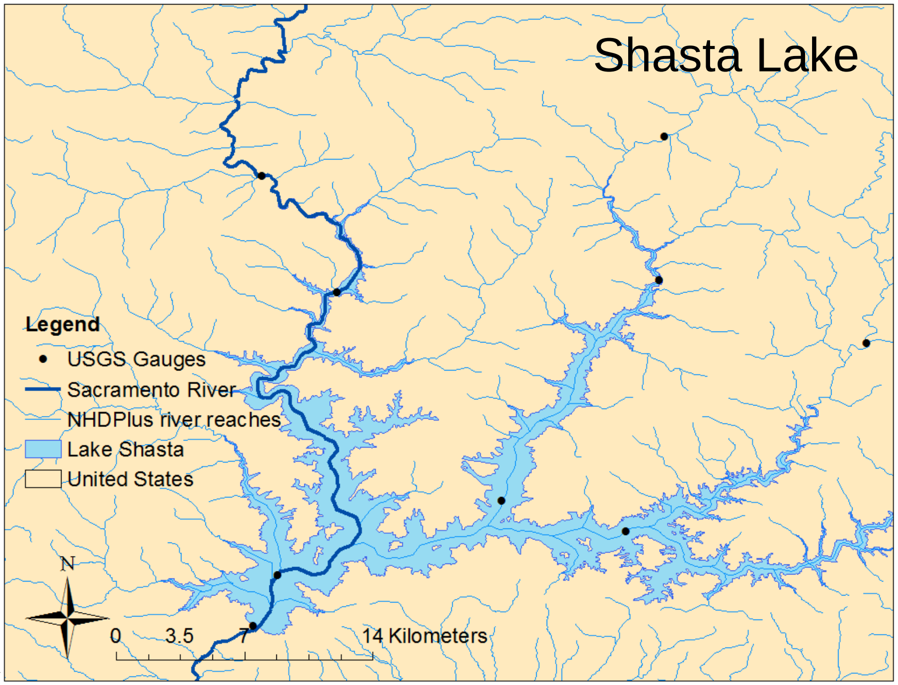



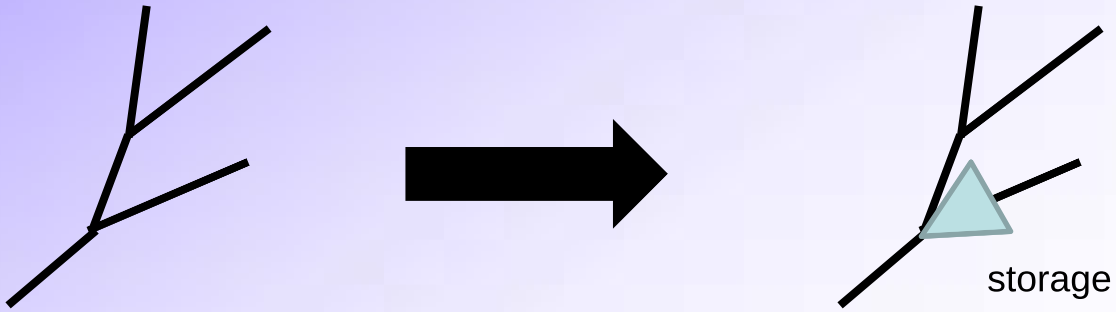

Shasta Lake

Daily flow for the year 2000, Shasta Lake

# Treatment for reservoirs is needed!!!



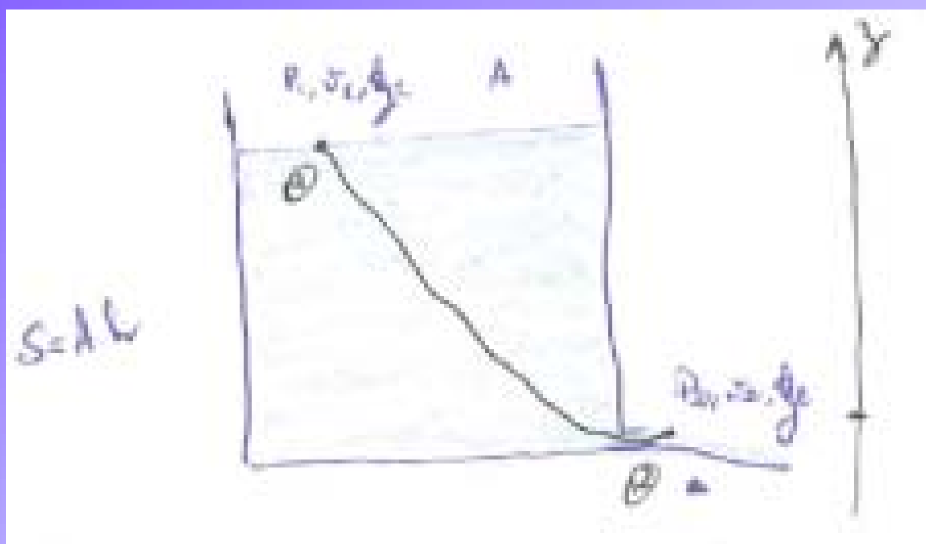
# Research motivations

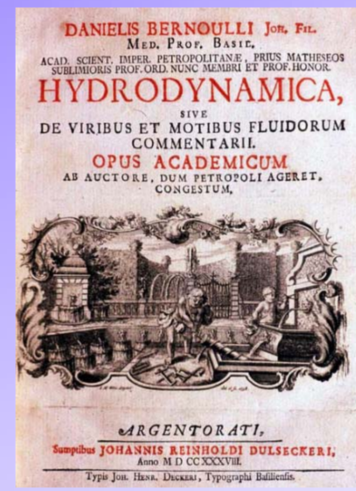
- Add a simple reservoir model to RAPID that is based on natural physics
  - Advantages:
    - First step to reservoir modeling
    - Get started on coupling rivers/reservoirs
  - Limitations:
    - Does not explicitly account for storage of water as water supply and for corresponding management practices

# Bernoulli Principle applied to reservoirs

$$\frac{v^2}{2} + g \cdot z + \frac{P}{\rho} = \text{constant}$$

*For incompressible and inviscid fluids*





DANIELIS BERNOULLI Joh. Fil.
Med. Prof. Basil.
ACAD. SCIENT. IMPER. PETROPOLITANÆ, PRIUS MATHESEOS SUBLIMIORIS PROF. ORD. NUNC MEMBRI ET PROF. HONOR.
HYDRODYNAMICA,
SIVE
DE VIRIBUS ET MOTIBUS FLUIDORUM COMMENTARII.
OPUS ACADEMICUM
AB AUCTORE, DUM PETROPOLI AGERET, CONGESTUM.
ARGENTORATI,
Sumptibus JOHANNIS REINHOLDI DULSECKERI,
Anno M D CCXXXVIII.
Typis Joh. Henr. Deckeri, Typographi Basiliensis.

1738


Daniel Bernoulli
(1700-1782)

*Assuming water height is almost constant*

$$\boxed{Q^{out} = \frac{1}{\tau} \cdot V} \qquad \tau = \frac{A}{a}\sqrt{\frac{h}{2 \cdot g}}$$

Linear reservoir equation

# Response of a linear reservoir

$$Q^{out} = \frac{1}{\tau} \cdot V$$

Linear reservoir equation

$$\frac{dV(t)}{dt} = Q^{in}(t) - Q^{out}(t)$$

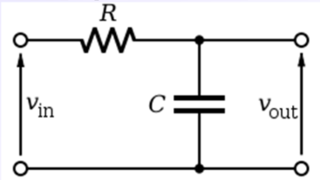
Continuity equation
(conservation of mass)

$$\frac{dQ^{out}(t)}{dt} + \frac{1}{\tau} \cdot Q^{out}(t) = \frac{1}{\tau} \cdot Q^{in}(t)$$

First order linear differential equation

Storage can be removed from the equation!!!

First order low-pass filter



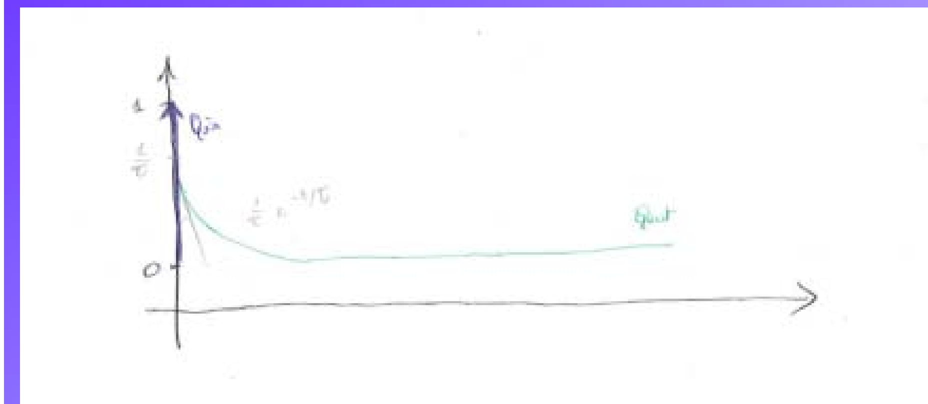
# Some review of existing literature

- Sherman, 1932, Streamflow from rainfall by the unit hydrograph method, *Eng. New-Record*
- Clark, 1945, Storage and the unit hydrograph
- Zoch, 1934, 1936, 1937, On the relation between rainfall and streamflow, *Monthly Weather Rev*.
- Nash, 1955, The relation of streamflow to rainfall, *ME Thesis, Univ College, Galway*
- Nash and Farrel, 1955, A graphical solution of the flood-routing equation for linear storage-discharge relation, *Trans. AGU*
- Dooge, 1956, Synthetic unit hydrographs based on triangular inflow, *MS thesis, Univ. Iowa*
- Nash, 1957, The form of the instantaneous unit hydrograph, *Int. Assoc. Hydrol. Sci. Gen. Assemb.*
- Nash, 1959, Systematic Determination of unit hydrograph parameters, *JGR*
- Nash, 1959, A note on the Muskingum flood routing method, *JGR*
- Dooge, 1959, A General Theory of the Unit Hydrograph, *JGR*

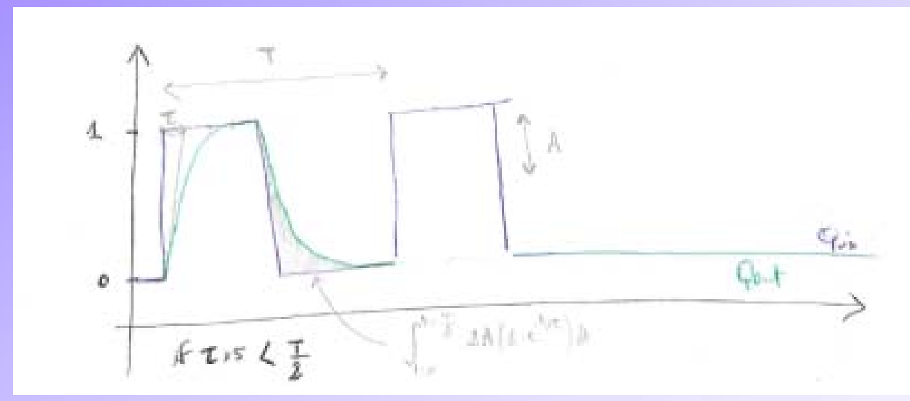
James Nash
(1927-2000)

James Dooge
(1922 – 2010)

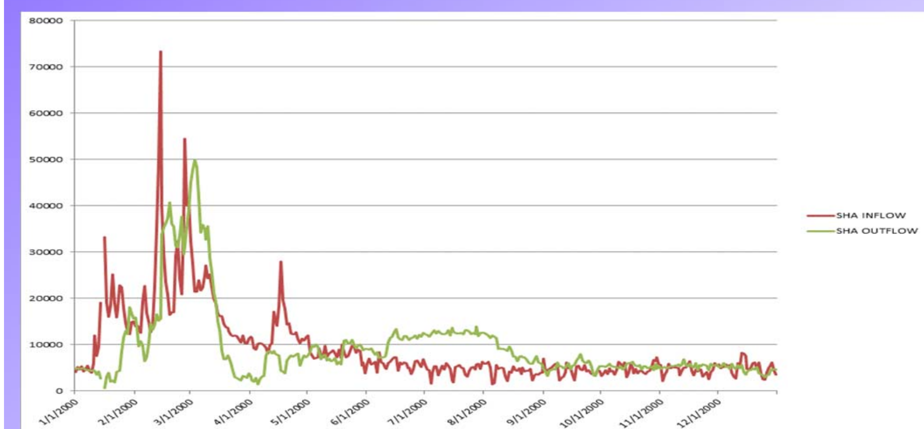
# Classic responses of low-pass filters



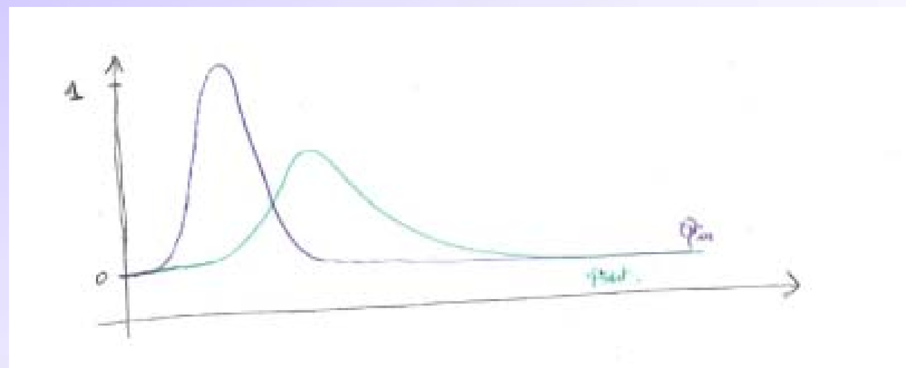
Unit impulse (Dirac delta function)



Square signal (or succession of unit step functions)




Flood wave???



What if we assume first half of sine wave ???

# Response of linear reservoir to sine wave (analytical solution)

$$\begin{cases} \dfrac{dQ^{out}(t)}{dt} + \dfrac{1}{\tau} \cdot Q^{out}(t) = \dfrac{1}{\tau} \cdot Q^{in}(t) \\ Q^{in}(t) = \sin\left(\dfrac{2\pi}{T} \cdot t\right) \\ Q^{out}(0) = 0 \end{cases}$$

Differential equation

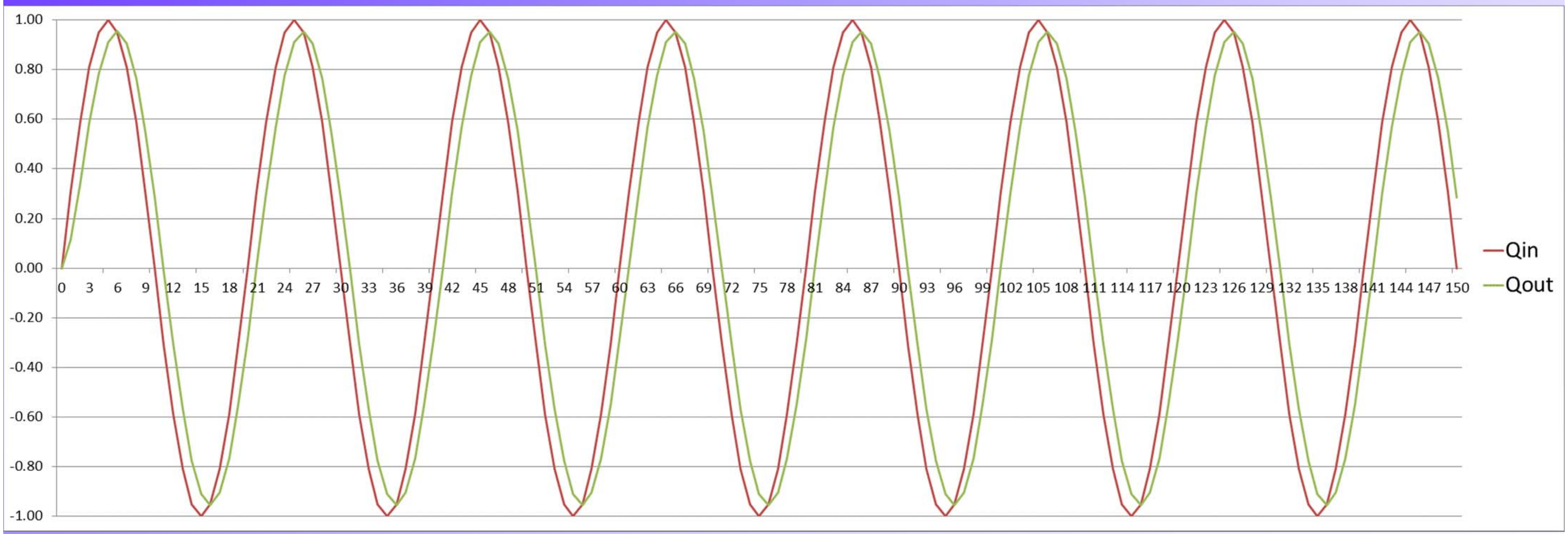
Inflow

Initial condition

$$\begin{cases} \boxed{Q^{out}(t) = \dfrac{1}{1+\left(\dfrac{2\pi\tau}{T}\right)^2} \cdot \left[\dfrac{2\pi\tau}{T} \cdot \exp\left(\dfrac{-1}{\tau} \cdot t\right) + \sqrt{1+\left(\dfrac{2\pi\tau}{T}\right)^2} \cdot \sin\left(\dfrac{2\pi}{T} \cdot \left(t - t_\varphi\right)\right)\right]} \\ t_\varphi = \dfrac{T}{2\pi} \cdot \arctan\left(\dfrac{2\pi\tau}{T}\right) \end{cases}$$

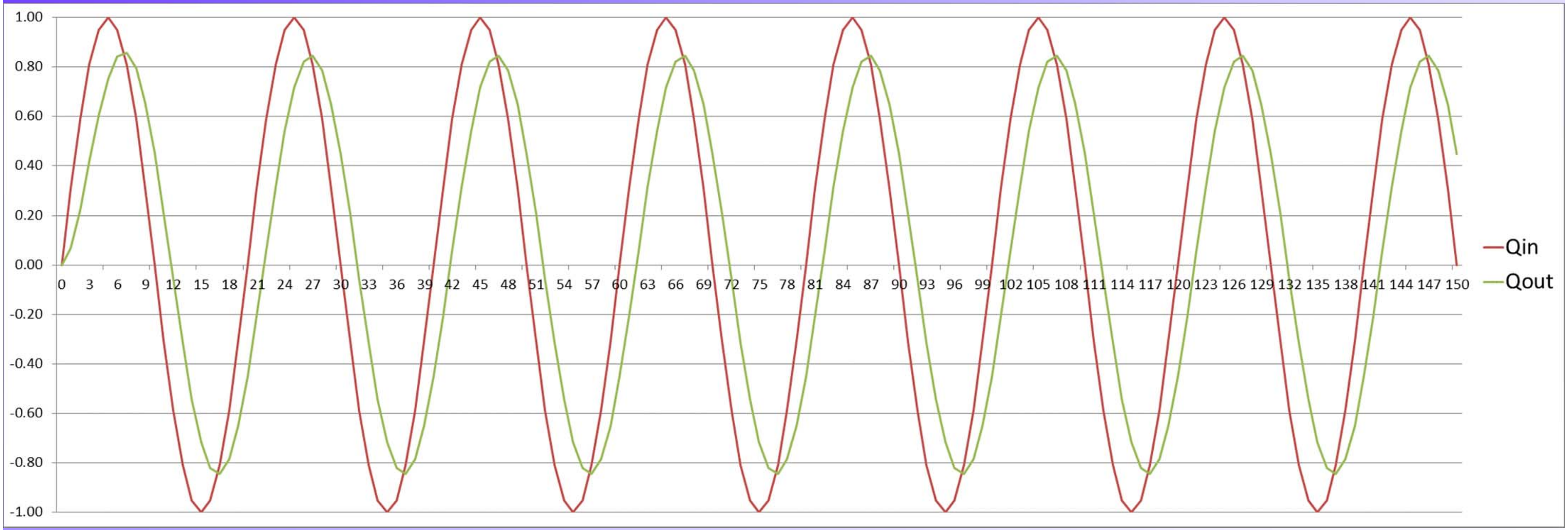
Exact solution

# Inflow and outflow vs time




T=20
τ=1

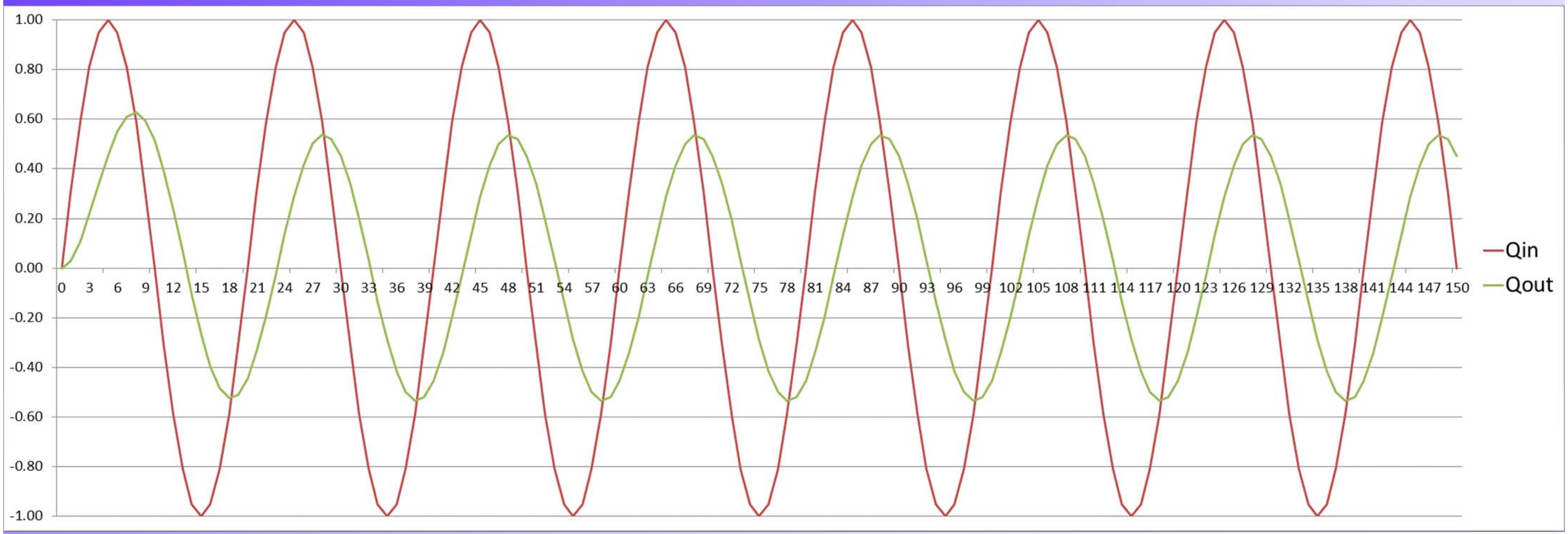
# Inflow and outflow vs time




T=20

τ=2

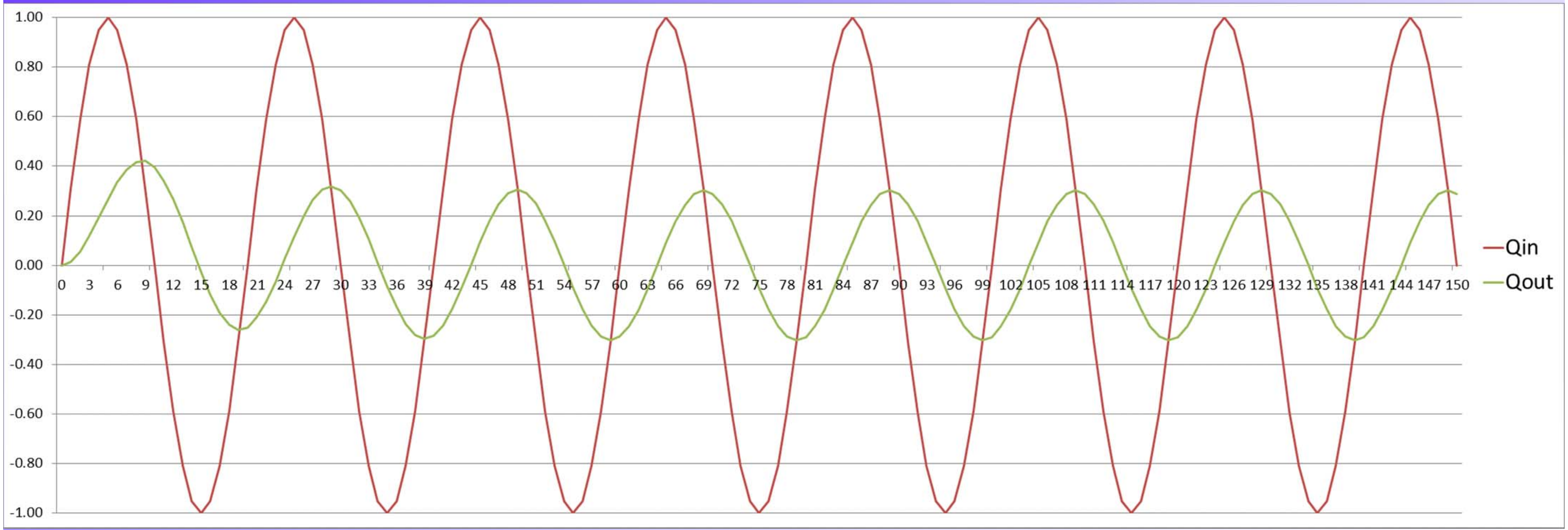
# Inflow and outflow vs time




T=20

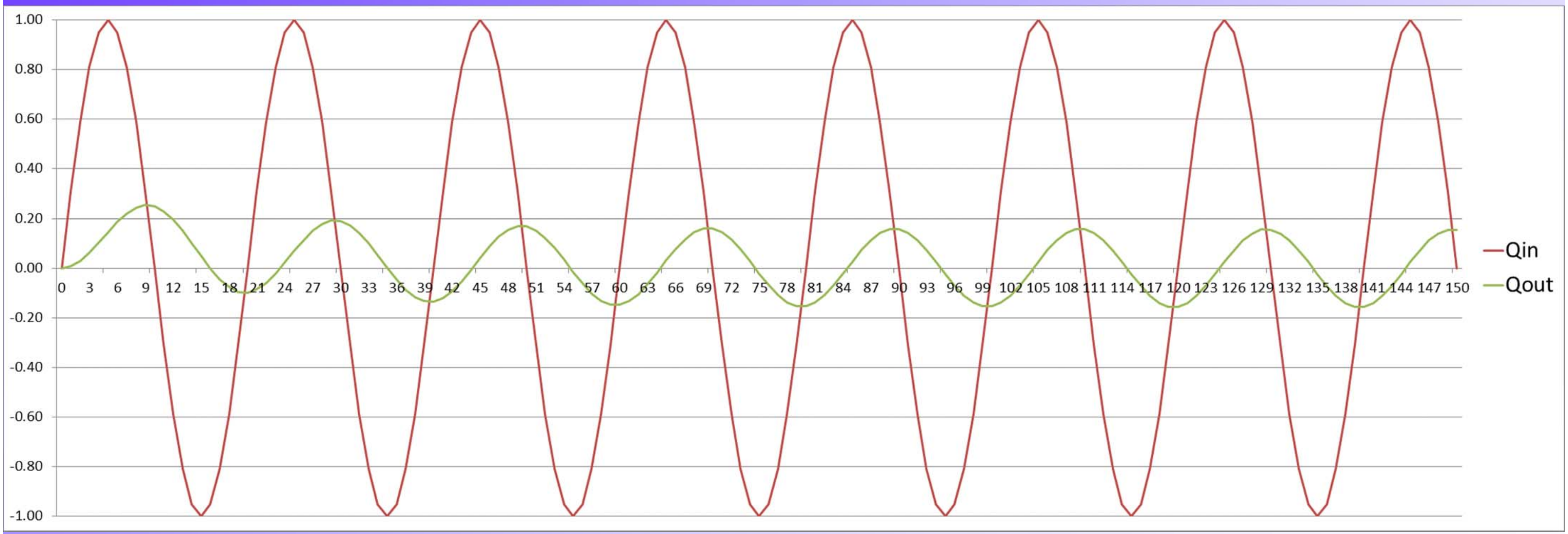
τ=5

# Inflow and outflow vs time




T=20
τ=10

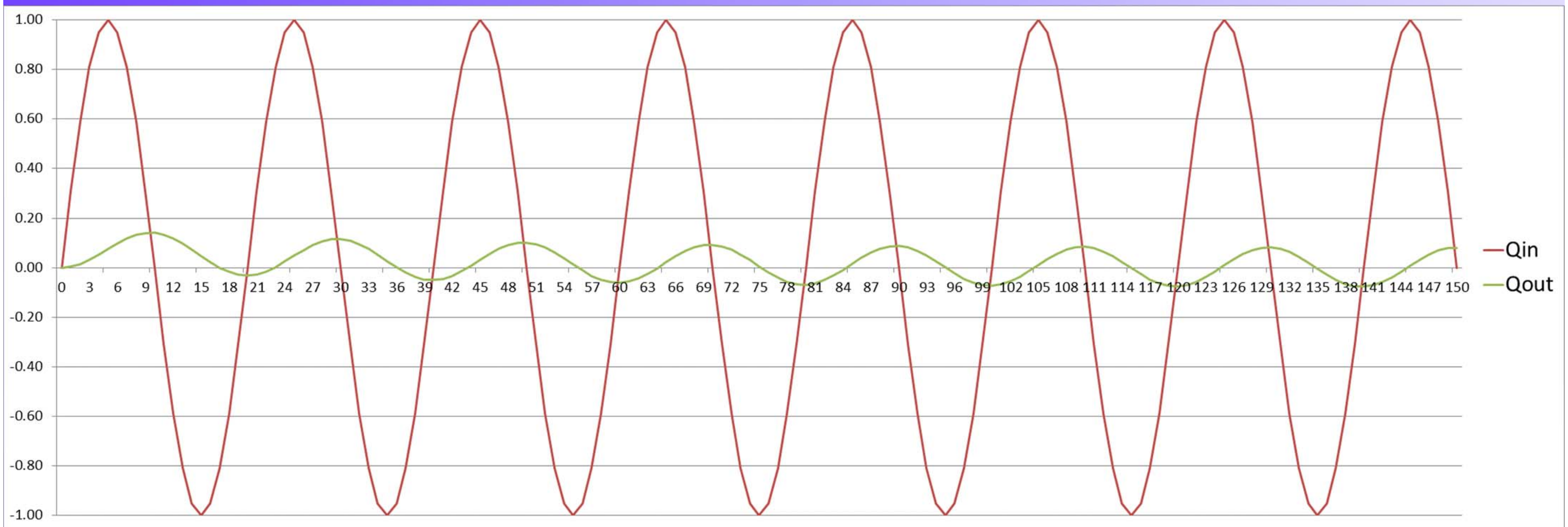
# Inflow and outflow vs time




T=20

τ=20

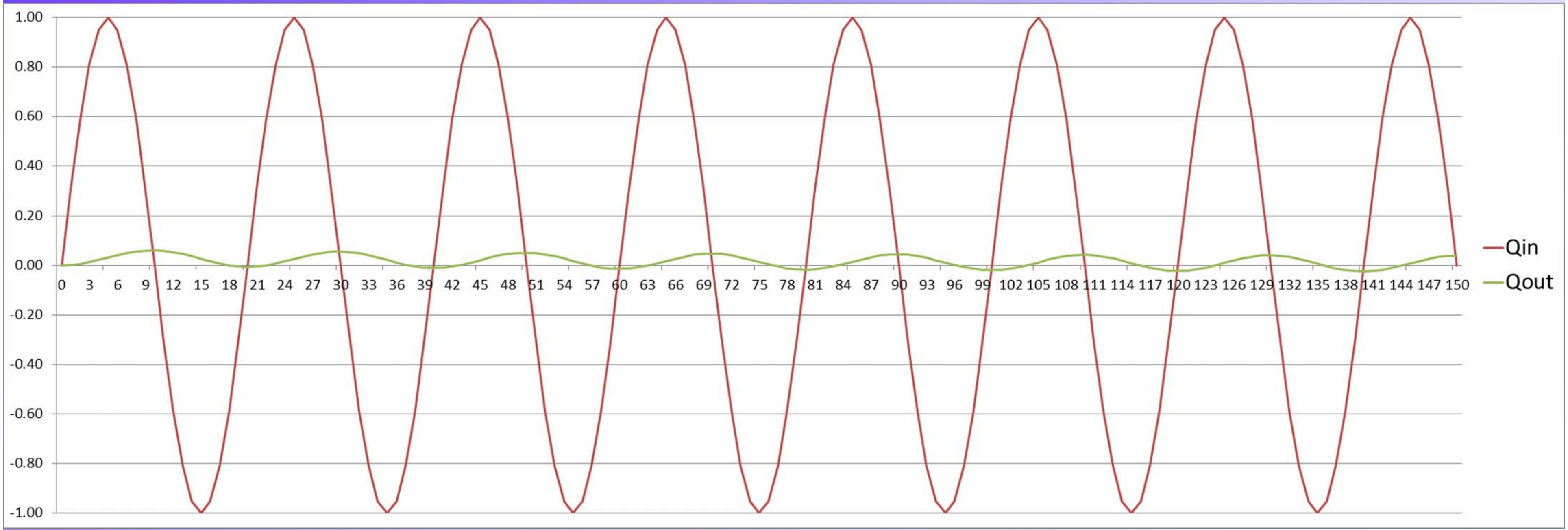
# Inflow and outflow vs time




T=20

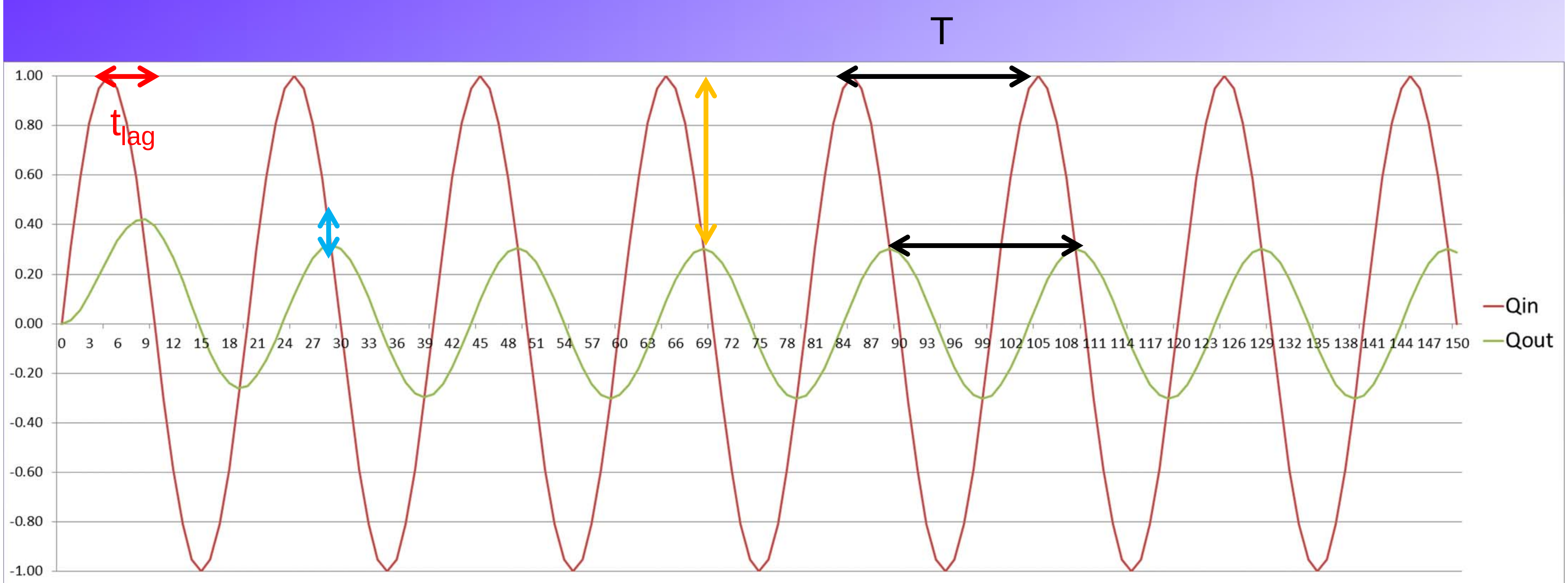
τ=40

# Inflow and outflow vs time




T=20
τ=100

# What did we observe???



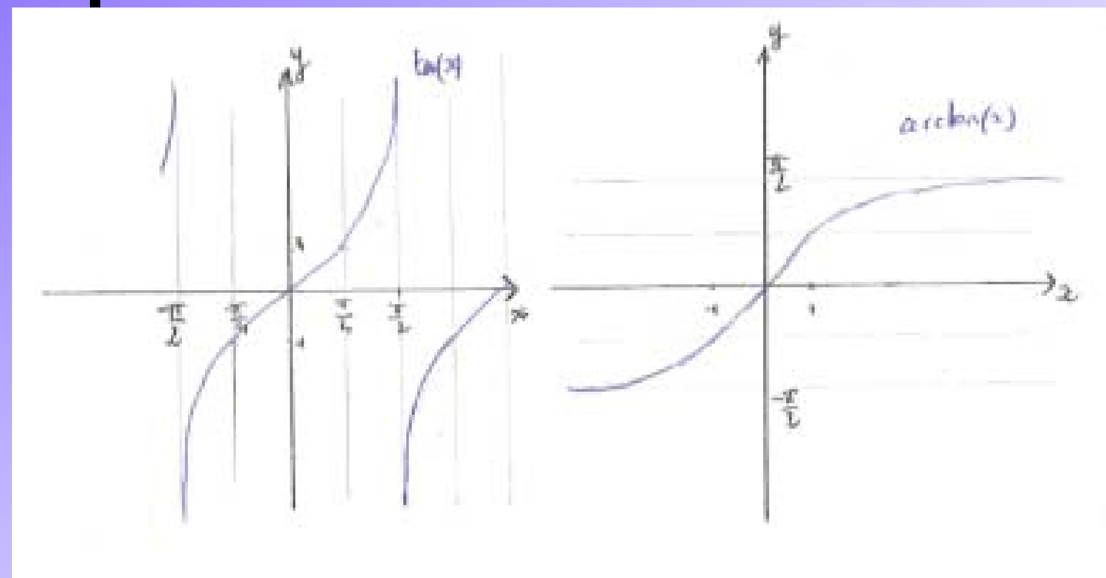

T=20
τ=10

# Some characteristics of these hydrographs

- The outflow seems to be a periodic signal of same period as inflow, but:
  - with a different phase (outflow delayed)
  - with a different magnitude (outflow smaller)
- The difference in phase seems to increase with the value of $\tau/T$ but reaches a maximum for large values of $\tau/T$
- The difference in magnitude seems to increase with the value of $\tau/T$
- The initial peak of the outflow is higher than the following peaks

# Quantification of the difference in phase

$$t_\varphi = \frac{T}{2\pi} \cdot \arctan\left(\frac{2\pi\tau}{T}\right)$$




$$\left.\begin{array}{l} T > 0 \\ \tau > 0 \end{array}\right\} t_\varphi \in \left]0, T/4\right[$$

$$\frac{\tau}{T} \square\ 1 \Rightarrow t_\varphi \square\ 0, t_\varphi > 0$$

$$\frac{\tau}{T} \square\ 1 \Rightarrow t_\varphi \square\ \frac{T}{4}, t_\varphi < \frac{T}{4}$$

$$\boxed{\tau = T \cdot \frac{1}{2\pi} \cdot \tan\left(\frac{2\pi}{T} \cdot t_{lag}\right)}$$

If $t_{lag}$ is close to T/4, (i.e. τ/T>>1) impossible to get an accurate estimate of τ

# Some maths…

$$\max_{t\in[0,T]}\left[Q^{out}(t)\right]=\frac{1}{1+\left(\frac{2\pi\tau}{T}\right)^2}\cdot\left[\frac{2\pi\tau}{T}\cdot\exp\left(\frac{-T}{2\tau}\right)+\sqrt{1+\left(\frac{2\pi\tau}{T}\right)^2}\right]$$

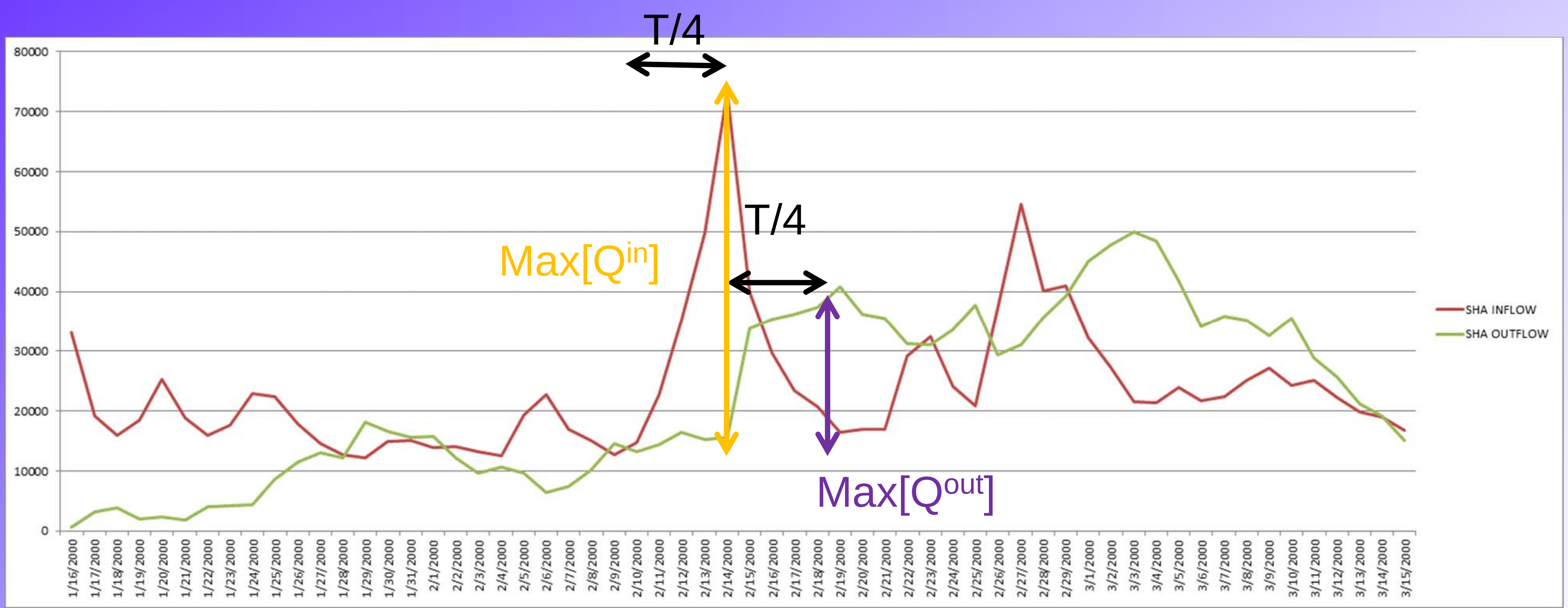
Brook Taylor
(1685-1731)

- Taylor's series expansion
- Resolution of a second order polynomial
- Assumption that *max[$Q^{out}$]* is small

$$\boxed{\frac{\tau}{T}=\frac{1}{\pi\cdot\max_{t\in[0,T]}\left[Q^{out}(t)\right]}-\frac{1}{4}}$$

Valid for $t_{lag}$ close to T/4
(i.e. $\tau$/T>>1)

# Application to Shasta Lake




Jan – Mar 2000, daily

Rising limb of $Q^{in}$=5 days
Rising limb of $Q^{out}$=5 days
$T_{lag}$=5 days
Max[$Q^{in}$]=73,332 cfs
Max[$Q^{out}$]=40,776 cfs

$$\tau = 4 \cdot 5 \cdot \frac{1}{\pi \cdot \frac{44776}{73332}} - \frac{1}{4} = 6.45 days$$

# Numerical approximation for linear reservoir

$$\frac{dQ^{out}(t)}{dt} = \frac{Q^{out}(t+\Delta t) - Q^{out}(t)}{\Delta t}$$

First order explicit Euler Method applied to Continuity Eq

$$Q^{out} = \frac{1}{\tau} \cdot V$$

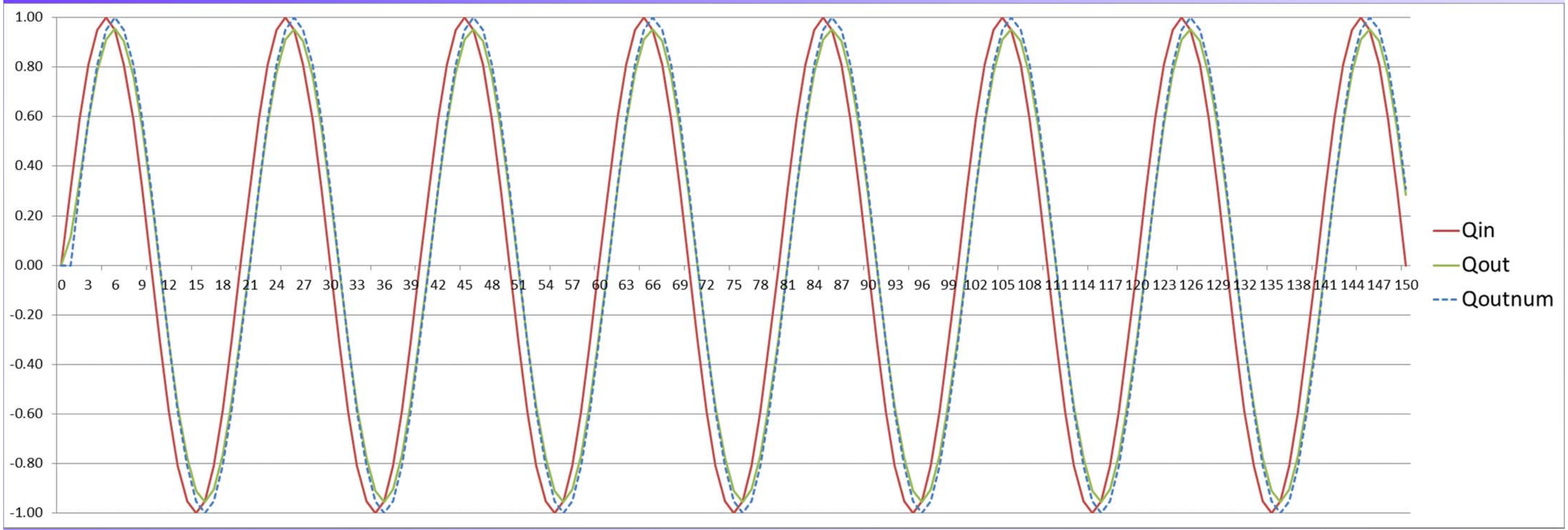
Linear reservoir Eq

Leonhard Euler (1707-1783)

$$Q^{out}(t+\Delta t) = \frac{\Delta t}{\tau} \cdot Q^{in}(t) + \left(1 - \frac{\Delta t}{\tau}\right) \cdot Q^{out}(t)$$

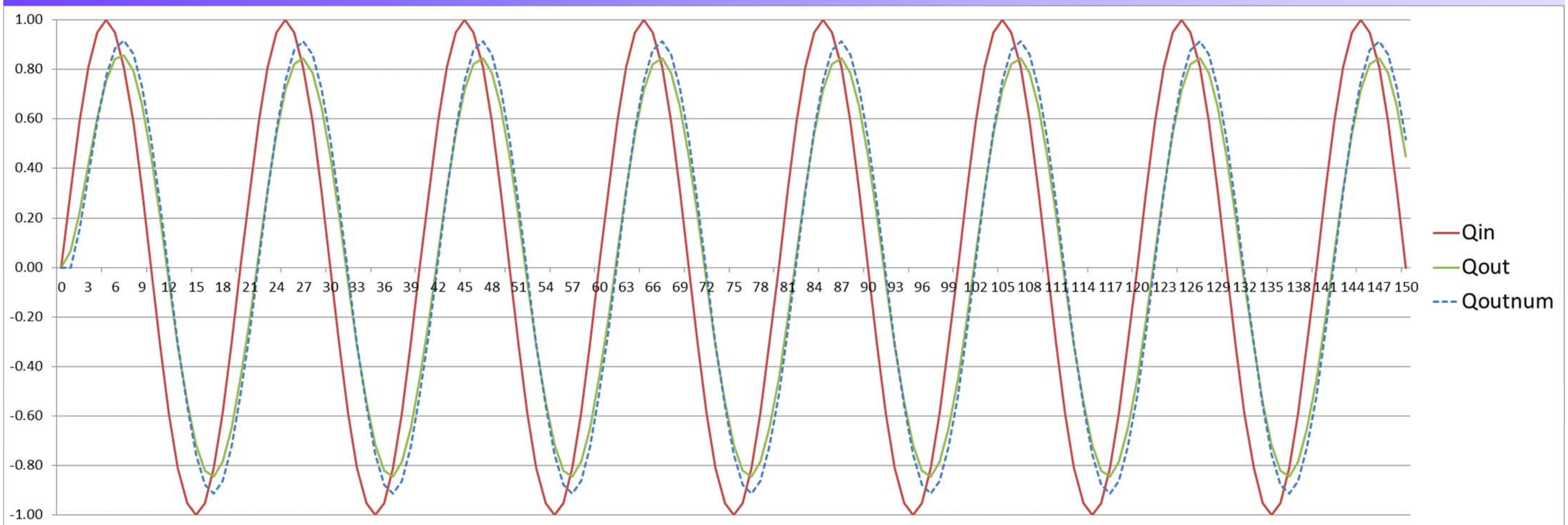
Numerical method for linear reservoir

# Inflow, outflow and numerical outflow vs time



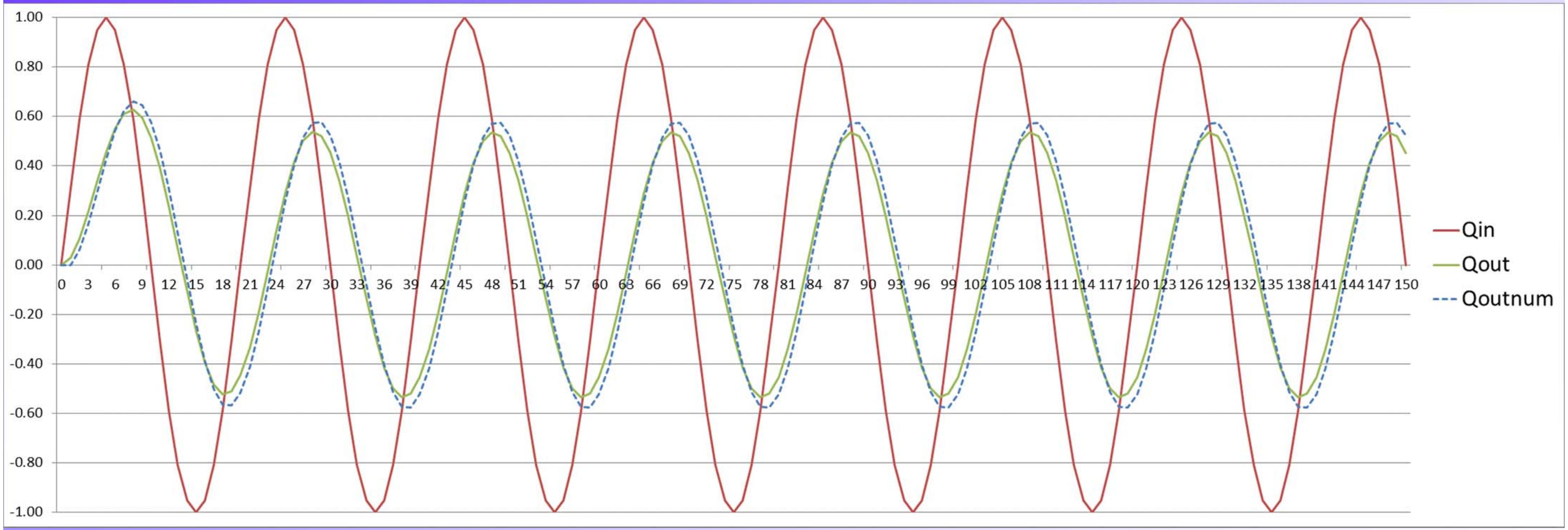

T=20
τ=1

# Inflow, outflow and numerical outflow vs time




T=20
τ=2

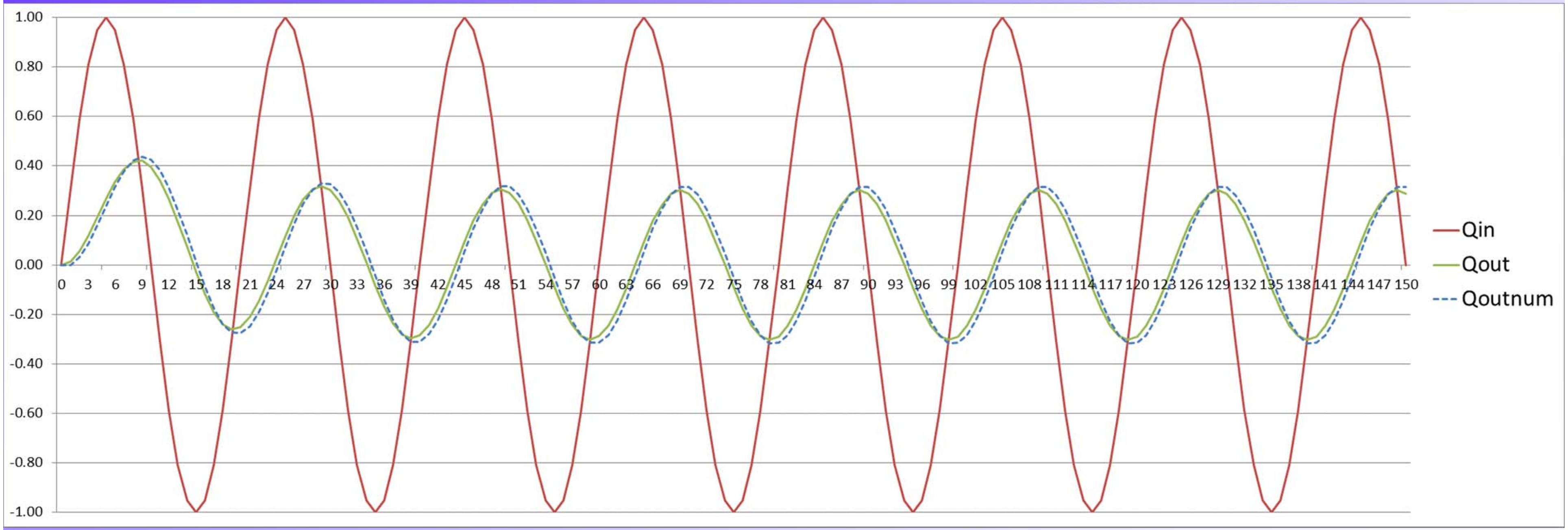
# Inflow, outflow and numerical outflow vs time




T=20

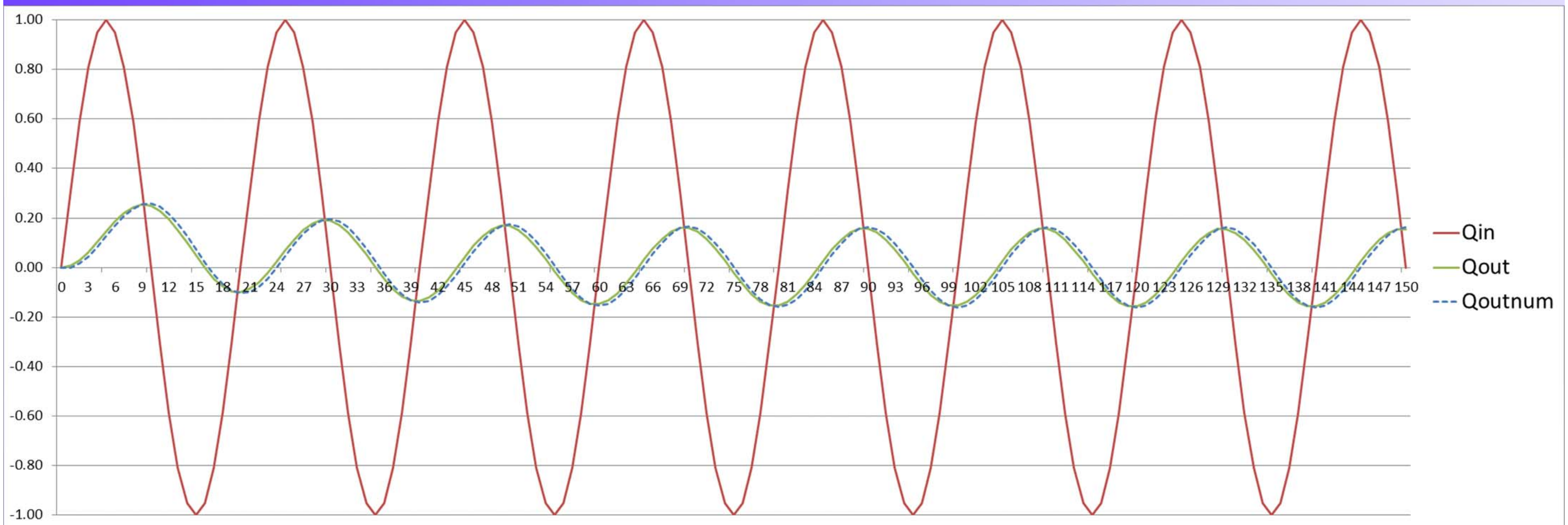
τ=5

# Inflow, outflow and numerical outflow vs time




T=20
τ=10

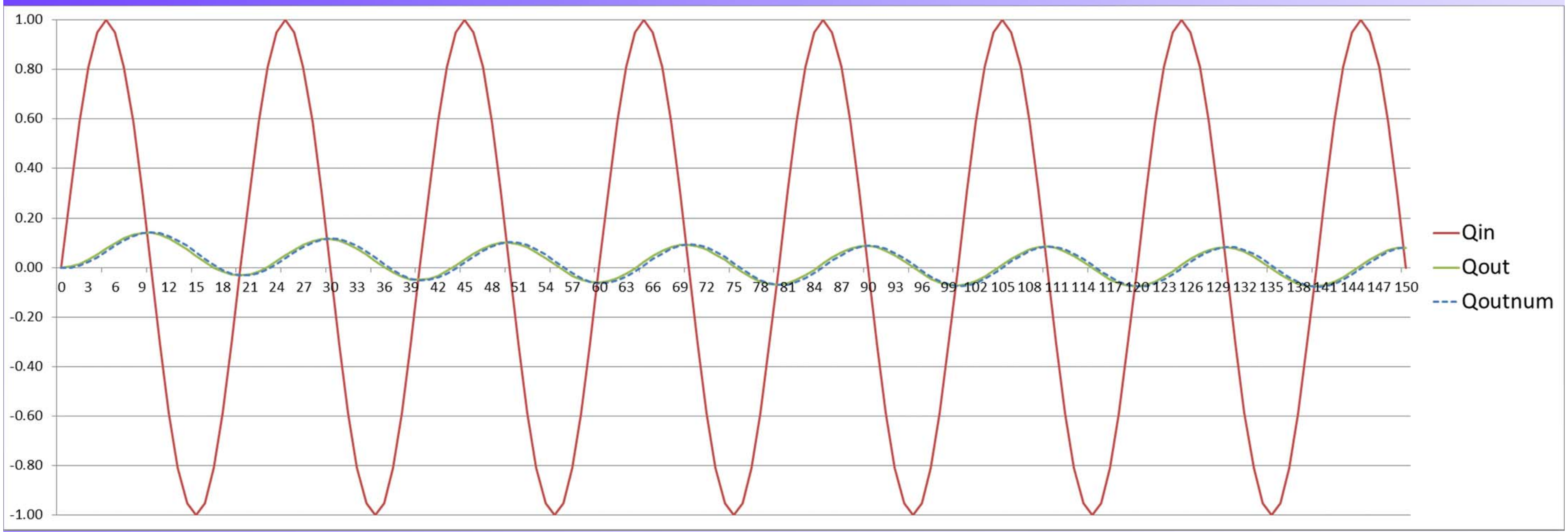
# Inflow, outflow and numerical outflow vs time




T=20

τ=20

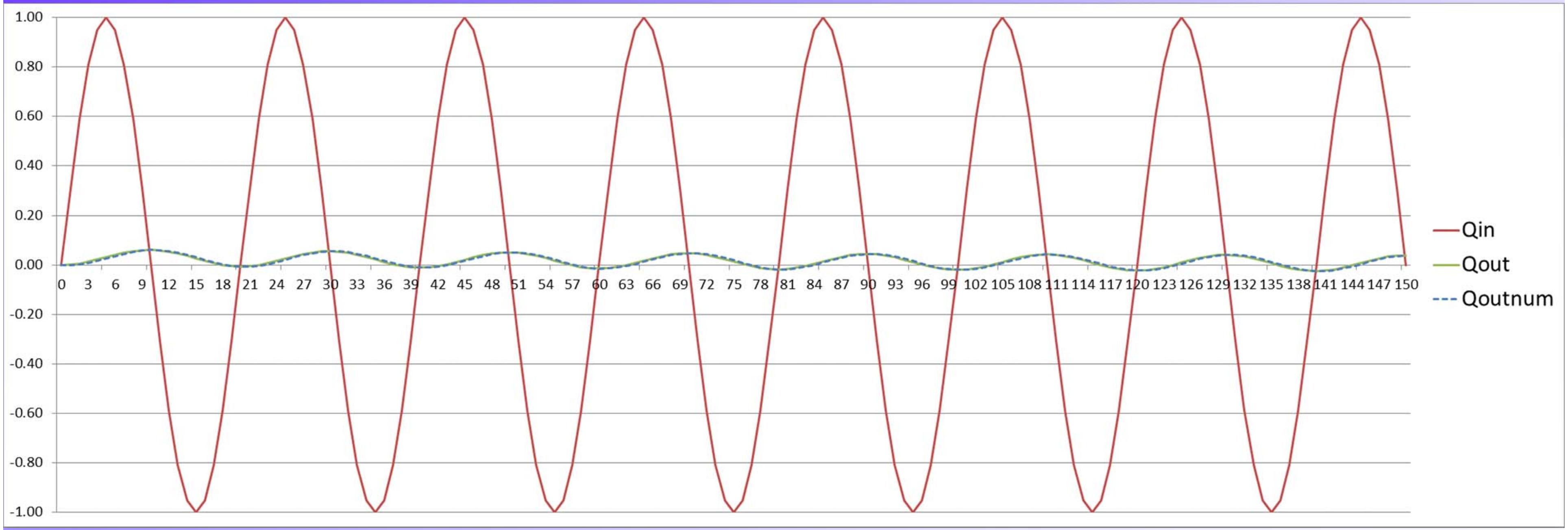
# Inflow, outflow and numerical outflow vs time




T=20

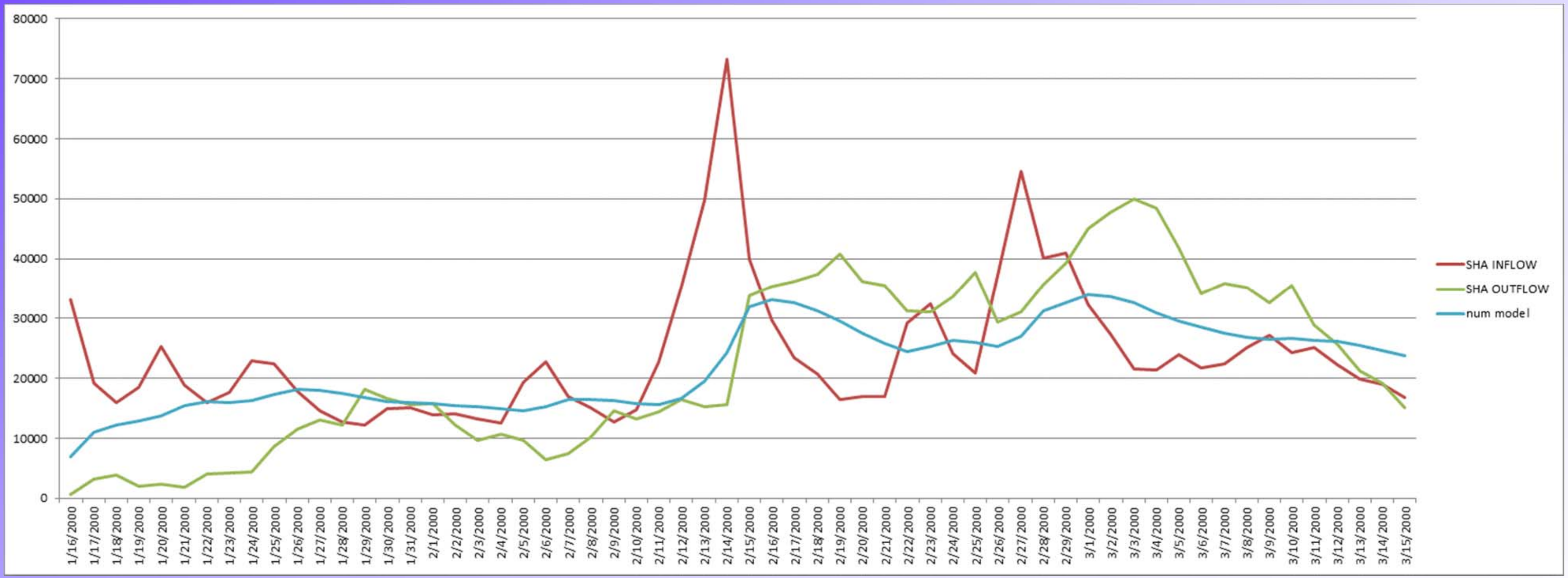
τ=40

# Inflow, outflow and numerical outflow vs time



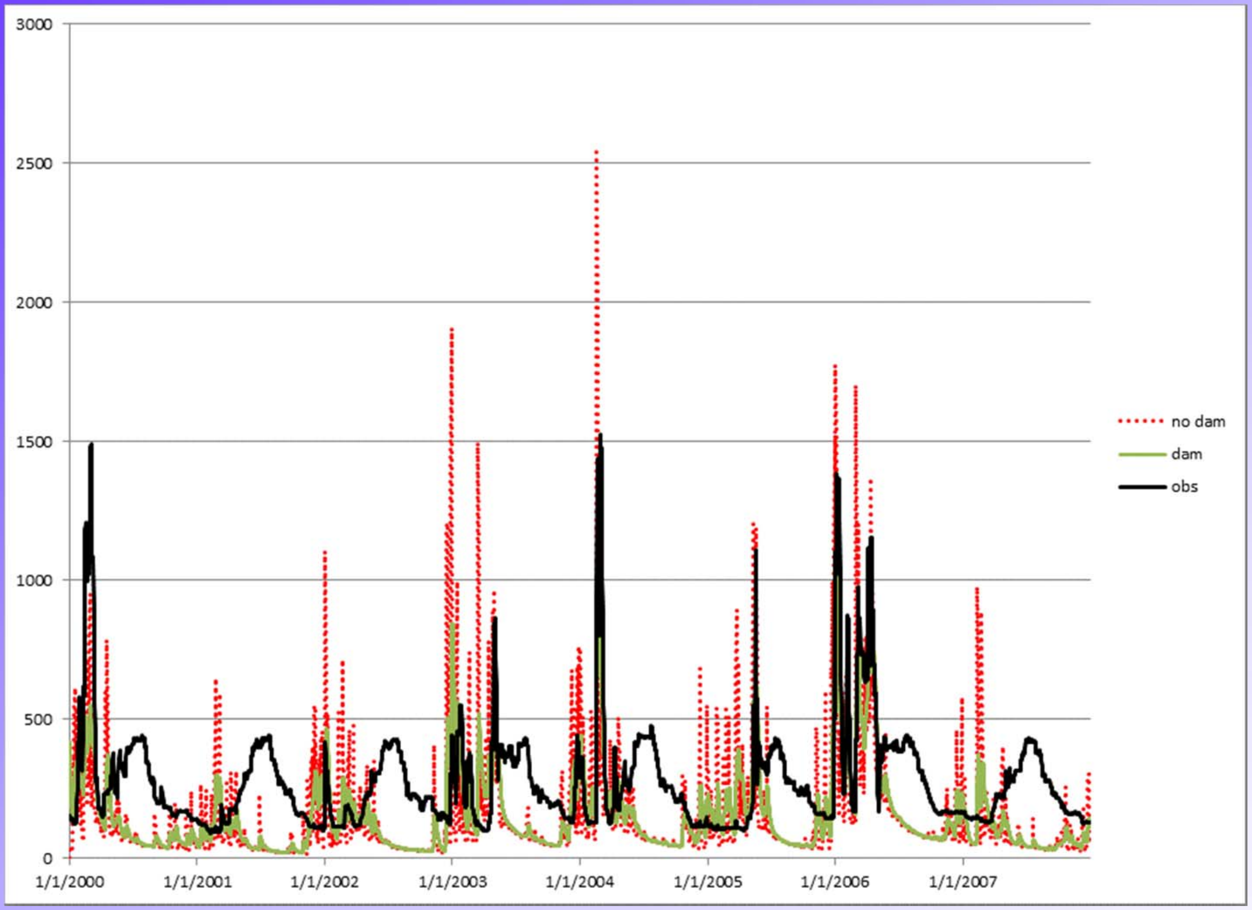

T=20
τ=100

# Reservoir modeling of Shasta Lake




Jan – Mar 2000, daily

# RAPID with and without linear reservoirs




Shasta Lake

Applied to the two biggest reservoirs in California.

RMSE and efficiency improved a the five corresponding downstream stations.

Results are statistically significant!!!

*Hydrological Sciences - Journal - des Sciences Hydrologiques*, **35**, 6, 12/1990

The education of hydrologists
(Report of an IAHS/UNESCO Panel on hydrological education)

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*Ralph M. Parsons Laboratory, Department of Civil Engineering, Building 48-335, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*

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W. H. VAN DER MOLEN
*Lindelaan 8, 6871 DX Renkum, The Netherlands*

# Thank you!

It is fashionable to regard man's involvement in nature as almost always bad. It is true that ignorance (often spurred on by greed) is leading to progressive damage to the natural environment and that we require as much scientific understanding as possible of the relevant processes in order to diagnose our mistakes and put them right. On a more positive note, however, man's ability to control his environment, to improve it and to make it more enjoyable, and indeed more productive and profitable, depends just as centrally on putting our understanding of hydrological processes on as sound a scientific basis as we can manage.

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