Generating a relative geologic time volume by 3D graph-cut phase unwrapping method with horizon and unconformity constraints

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ABSTRACT

Construction of a relative geologic time (RGT) volume is vital to seismic geomorphological and sedimentological interpretation. Seismic instantaneous phase unwrapping provides an excellent approach for generating an RGT volume. Although several 2D or 3D seismic phase unwrapping results have been published, there is a clear need for discussions on concrete methods for seismic phase unwrapping. We have developed the graph-cut phase unwrapping method, which performs well in the interferometric synthetic aperture radar image processing. It has advantages of strong discontinuity-preserving ability and high computing efficiency. To make it suitable for 3D seismic phase unwrapping, the method is improved by extending it from 2D to 3D, and by introducing the seismic horizon and unconformity constraints. The strong and continuous conformable seismic events, which can be easily tracked by certain autopicking methods, are introduced as horizon constraints for guiding the phase unwrapping to ensure a constant unwrapped phase on a constraining horizon. This idea is based on the fact that continuous seismic horizons are of time-stratigraphic significance. The horizon constraints can promise a correct unwrapped result on the constraining horizons and avoid the possible phase unwrapping errors propagating across a horizon. An unconformity represents a geologic time discontinuity, which is difficult to recover in an RGT volume by phase unwrapping. What’s worse, incorrect phase unwrapping on an unconformity will result in some discontinuities of unwrapped phase in the conformable data areas outside the unconformity. Interpreted unconformities are used as unconformity constraints to recover the discontinuities of the unwrapped phase at the constraining unconformities. As a test, our improved 3D graph-cut phase unwrapping method is successfully applied to the late Permian to early Triassic carbonate reservoirs in northern Sichuan Basin, southwest China. The results match well with the regional geologic background.

INTRODUCTION

The “Wheeler diagram” (Wheeler, 1958), also called the “chronostratigraphic section” (Vail et al., 1977), is an important tool for seismic stratigraphic interpretation to portray the time-space distribution of strata, and the related depositional, nondepositional, and erosional events. In the 3D case, a chronostratigraphic or Wheeler diagram represents a volume, namely the chronostratigraphic or Wheeler volume. Several authors tried to generate a Wheeler volume by flattening the corresponding seismic volume with or without horizon picking (e.g., Nordlund and Griffiths, 1993a, 1993b; Zeng et al., 1998a, 1998b; Lomask, 2003; Lomask et al., 2006; Stark, 2005a; De Groot et al., 2006; De Bruin et al., 2007; Fomel, 2010; Parks, 2010; Luo and Hale, 2011).

Stark (2003, 2005c) demonstrated the concept and generation of a relative geologic time (RGT) volume. An RGT volume is an attribute volume in which each sample is assigned a relative geologic time value to correspond to the reflection amplitude of that sample. The generation of an RGT volume is an important step toward the construction of a Wheeler volume.

An RGT volume can be generated in several ways (Stark, 2005a). A simple but laborious way is to pick out as many horizons as possible using standard manual or automatic tracking techniques, and then assign each tracked horizon a relative geologic time. Thus, the resulting RGT volume is actually composed of a set of discrete geologic time surfaces corresponding to the tracked horizons, and the value for each sample between the surfaces is assigned by an interpolation algorithm. It is obvious that the resolution of such an RGT...
volume thus depends on the accuracy of horizon tracking and the number of horizons tracked. The main problem of the method is that vast information between the horizons goes ignored, which reduces the resolution of the resultant RGT volume.

Stark (2003, 2004a, 2004b, 2005a, 2005b, 2005c, 2006) illustrates an excellent idea to generate an RGT volume through unwrapping seismic instantaneous phase. It is based on the fact that the phase attribute in seismic data carries time information in the process of wave propagation, that is, the absolute or true phase of seismic data always increases with traveltime; therefore, the absolute phase estimated by unwrapping the seismic instantaneous phase should be equivalent to the relative geologic time under the assumption that an event or a horizon with the same instantaneous phase corresponds to a geologically isochronal surface (Stark, 2003, 2005c).

Phase unwrapping is a common technique in certain fields of image processing, including the interferometric synthetic aperture radar imaging (InSAR), magnetic resonance imaging (MRI), and so on. Some phase unwrapping methods have been developed (e.g., Goldstein et al., 1988; Ghiglia and Pritt, 1998), but most of them are 2D, which is not suitable for phase unwrapping of 3D seismic data. In addition, due to the particularity of seismic phase unwrapping, the methods are expected to undergo certain modifications so that they can produce better results in seismic phase unwrapping. In particular, some geologic and geophysical conditions should be met in seismic instantaneous phase unwrapping. For example, the estimated unwrapped phase should always increase with traveltime in vertical seismic sections (Stark, 2003, 2005a), and an abrupt jump of the unwrapped phase should be expected at an unconformity, which is similar to a surface fault in InSAR data.

Shatilo (1992) discusses the methods of 1D seismic instantaneous phase unwrapping. Several authors report their 2D or 3D results of seismic instantaneous phase unwrapping (Stark, 2003, 2004b, 2005c; Imhof and Tech, 2004; De Matos et al., 2009), but no detailed discussion on the phase unwrapping method or algorithm was published. In this paper, the graph-cut phase unwrapping method, presented by Valadão and Bioucas-Dias (2009) in InSAR data processing, is introduced into seismic instantaneous phase unwrapping. Some improvements to the method are developed, including the extension of the method from 2D to 3D, and the introduction of geologic horizon and unconformity constraints into the process of phase unwrapping. As a test, our improved method is applied to the late Permian to early Triassic carbonate reservoirs in the northern Sichuan Basin, southwest China. The results match well with the regional geologic background, suggesting the robustness of the method.

**METHOD**

**Brief review of current phase unwrapping methods**

In general, the phase extracted from a real signal by a certain mathematical operation is referred to as a wrapped phase, which lies between $-\pi$ and $+\pi$ in radians. In reflection seismology, the instantaneous phase, extracted from seismic data through the Hilbert transform (Taner et al., 1979) or wavelet transform (e.g., Gao et al., 1999), belongs to wrapped phase $\varphi_i$. It is related to absolute phase $\phi_i$, the true phase of an actual signal, by

$$\varphi_i = \mathcal{W}(\phi_i) = \phi_i - 2\pi k_i,$$

where $k_i$ is an integer, and $\mathcal{W}$ is called the wrapped operator that wraps all values of its argument into the range $(-\pi, \pi]$ by adding or subtracting an integral multiple of $2\pi$ radians from its argument (Ghiglia and Pritt, 1998). Phase unwrapping is the operation to solve for the absolute phase $\phi_i$ from the wrapped phase $\varphi_i$. Define the absolute phase difference $\Delta \phi_i$ and the wrapped phase difference $\Delta \varphi_i$ as $\Delta \phi_i = \phi_i - \phi_{i-1}$ and $\Delta \varphi_i = \varphi_i - \varphi_{i-1}$, respectively. If the absolute phase satisfies the Itoh condition (Itoh, 1982),

$$|\Delta \phi_i| \leq \pi,$$

then

$$\Delta \varphi_i = \mathcal{W}(\Delta \phi_i).$$

This suggests that phase unwrapping is, theoretically, a simple line integration problem over the wrapped phase data, assuming the absolute phase $\phi_i$ satisfies the Itoh condition (equation 2). The absolute phase at the arbitrary $n$th point of the integration or phase unwrapping path can be calculated, starting at point $i_0$, by

$$\phi_n = \varphi_{i_0} + \sum_{i=1}^{n} \Delta \varphi_i.$$

Unfortunately, the Itoh condition is not always satisfied, due to the existence of noise, objective discontinuities (e.g., unconformities or faults in seismic data), and possible undersampling (i.e., violations of Shannon’s law) in real wrapped phase data. Therefore, instead of a simple line integration problem, phase unwrapping is usually a complicated ill-posed problem (Valadão and Bioucas-Dias, 2009).

Numerous phase unwrapping methods have been developed, which can be roughly grouped into four types: the path-following methods, the minimum-norm methods, the Bayesian methods, and the parametric-model-based methods (Dias and Leitão, 2002). In the path-following methods, phase unwrapping is achieved by integrating over the whole wrapped phase data along a given integration path, on which all the corresponding absolute phase data are assumed to meet the Itoh condition. Knowing how to find a correct integration path is crucial to the methods. Two main methods have been presented, which are the branch-cut method and quality-guided method, respectively. The branch-cut method (Goldstein et al., 1988) tries to find the samples that violate the Itoh condition and then connect them by lines, the so-called “branch cuts,” which will not be passed by the phase unwrapping path. Although it runs fast and saves memory, the branch-cut method uses only the information from local phase points in connecting the branch cuts. Once the branch cuts are connected incorrectly, errors will propagate along the phase unwrapping path, which means the phase unwrapping result is not a globally optimized solution. Moreover, due to the influences of possible unconformities, faults, and noise, large amounts of samples violating the Itoh condition may exist in real seismic instantaneous phase data, which may make the correct placing of branch cuts in seismic phase unwrapping difficult. The quality-guided method locates the points that violate the Itoh condition by using a quality map, which is generally extracted from a correlation map or a phase derivative variance map of the wrapped phase.
data. In such a quality map, the points that violate the Itoh condition tend to have low values (Ghiglia and Pritt, 1998). Therefore, an integration path is suggested to follow the points with high values in the quality map, and avoid those with low values. If a reliable quality map is available, the method usually performs better than the branch-cut method (Ghiglia and Pritt, 1998).

Minimum-norm phase unwrapping methods are based on the assumption that most absolute phase values satisfy the Itoh condition, and thus the absolute phase difference is almost always equal to the wrapped phase difference (equation 3). The methods are used to find an absolute phase solution when the \( L^p \) norm of the difference between the absolute phase difference and the wrapped phase difference is minimized. Therefore, the minimum-norm methods adopt a global optimization strategy to solve for the unwrapped phase (Valadão, 2006). The main problem of these methods is that they essentially neglect the absolute phase samples that do not meet the Itoh condition, including those that represent the true discontinuities of the interested geologic objects (for example, the stratal unconformities or faults in the seismic data). When \( p = 2 \), phase unwrapping turns into a classical least-squares problem, from which a unique global optimum solution can be obtained by the fast Fourier transform technique. In this case, however, the true discontinuities of the absolute phase will be smoothed without the constraints from a quality map. The representative methods for the case \( p = 1 \) include Flynn’s (1997) minimum weighted discontinuity algorithm and Costantini’s (1998) network programming method. Superior to the least-squares methods when \( p = 2 \), both methods have good discontinuity-preserving properties. When \( 0 \leq p < 1 \), the discontinuity-preserving ability of the algorithms is further enhanced, but the algorithms will become extraordinarily complex and they turn into NP-hard problems (Bioucas-Dias and Valadão, 2007).

The Bayesian methods (e.g., Dias and Leitão, 2002; Bioucas-Dias and Valadão, 2005, 2007; Valadão, 2006; Valadão and Bioucas-Dias, 2009) treat phase unwrapping as a problem of “maximum a posteriori” (MAP) estimation. According to the Bayesian theorem, the MAP estimation can be transformed to a \( L^p \) norm minimization of an energy function, by introducing the a priori probability and the likelihood function. The a priori probability can be acquired by using a first-order Gauss-Markov random field to model the absolute phase, and the likelihood function can be constructed based on a data-observation mechanism model (Bioucas-Dias and Valadão, 2007; Dias and Leitão, 2002). The methods have good denoising ability because noise is considered in the construction of the observation mechanism model, which makes the methods superior to the path following and minimum-norm methods mentioned above. Bioucas-Dias and Valadão (2005, 2007); Valadão (2006); and Valadão and Bioucas-Dias (2009) applied the graph-cut method to solve the \( L^p \) norm minimization problem. The graph-cut method is based on the max-flow/min-cut algorithms (Greig et al., 1989; Veksler, 1999; Boykov et al., 2001; Boykov and Kolmogorov, 2004; Kolmogorov and Zabih, 2004); and, with high flexibility, it is suitable for solving the \( L^p \) norm minimization for \( 0 \leq p < 1 \). The method is characterized by good denoising performance, good discontinuity-preserving ability (when \( 0 \leq p < 1 \)), and high efficiency of computation.

For the methods based on parametric models (Friedlander and Francos, 1996; Liang, 1996), absolute phase is solved by fitting the estimated absolute phase to a given parametric model. If a reasonable parametric model is available, a globally optimal solution of phase unwrapping result will be obtained. However, a parametric model matching well with the whole absolute phase data is usually difficult to find, which limits the applications of the methods (Dias and Leitão, 2002).

**Method adopted in this paper**

On the basis of a comparative analysis of the phase unwrapping methods mentioned above, the graph-cut phase unwrapping method, presented by Valadão and Bioucas-Dias (2009), is introduced into the seismic instantaneous phase unwrapping to generate an RGT Volume. To make the method suitable for seismic phase unwrapping, some improvements have been developed, including the extension of the original 2D method into a 3D one, and the introduction of the seismic horizon and unconformity constraints into the process of phase unwrapping. These improvements greatly increase the accuracy and reliability of seismic phase unwrapping results.

As one type of Bayesian method, the graph-cut method is built on a statistical model in which the absolute phase \( \phi_i \) and the wrapped phase \( \psi_i \) are considered as random variables, and the sets of the phases are treated as random fields, which are denoted as \( \Phi = \{\phi_i, i \in V\} \) and \( \Psi = \{\psi_i, i \in V\} \), respectively. \( V \) represents the set of all the data samples of the corresponding data set. Then the phase unwrapping from \( \Psi \) to \( \Phi \) can be regarded as an estimation problem using the MAP estimator:

\[
\Phi = \arg \max_\Phi p(\Phi|\Psi) \equiv \arg \max_\Phi p(\Psi|\Phi) p(\Phi),
\]

where \( p(\Psi|\Phi) \) is the likelihood function; \( p(\Phi|\Psi) \) and \( p(\Phi) \) are the posterior and a priori probabilities, respectively.

**Observation model and likelihood function**

Define a seismic data set \( s = \{s_i, i \in V\} \), which is considered as a random field. A seismic observation model can be depicted as

\[
s_i = a_i \cos \phi_i + n_i,
\]

where \( a_i \) and \( \phi_i \) are instantaneous amplitude and absolute phase of the theoretical seismic data without noise, respectively; \( n_i \) represents noise. By applying the Hilbert transform to equation 6, the corresponding complex seismic data \( S_i \) are obtained, which can be expressed as

\[
S_i = A_i e^{-j\phi_i} + N_i = |S_i| e^{-j(j\phi_i + \phi_{n_i})} = |S_i| e^{-j(j\phi_{n_i})},
\]

where \( N_i \) is the complex noise corresponding to \( n_i \); \( \phi_{n_i} \) is the phase from the complex noise \( N_i \), and \( \phi_{n_i} \) is the absolute phase of the complex seismic data \( S_i \) with noise; \( j \) is the imaginary unit.

Assume the noise \( N_i \) meets \( N_i \sim (0, \sigma^2) \) and is independent to the absolute phase \( \phi_i \), then the likelihood function of \( S_i \) can be written by (Valadão and Bioucas-Dias, 2009)

\[
p(S_i|\phi_i) = \frac{1}{\pi \sigma^2} e^{-\frac{(S_i - e^{-j\phi_i})^2}{\sigma^2}}.
\]

This can be simplified as

\[
p(S_i|\phi_i) \propto c e^{\lambda_i \cos(\phi_i - \phi_n)},
\]
where $c$ is a constant; $\lambda_i = \frac{24|x_k|}{\epsilon}$; and $\varphi_i = \arg(S_i) = \mathcal{W}(\phi_{S_i})$ ($\phi_i \in [-\pi, +\pi]$) is the wrapped instantaneous phase computed from the observed seismic data with noise. Therefore, the likelihood function of $\varphi_i$ can be written by

$$p(\varphi_i | \Phi) \propto c e^{\lambda_i \cos(\varphi_i - \varphi_i)}.$$  \hfill(10)

Assuming the components of $\Psi$ are conditionally independent, then

$$p(\Psi | \Phi) = \prod_{i \in \mathcal{E}} p(\varphi_i | \Phi) \propto c e^{\sum_{i,j \in \mathcal{E}} \lambda_i \cos(\varphi_i - \varphi_j)}.$$ \hfill(11)

A priori probability

Considering the absolute phase data set $\Phi$ as a first-order Gauss-Markov random field, the a priori probability can be given by (Dias and Leitão, 2002)

$$p(\Phi) \propto \exp\left(-\mu \sum_{i,j \in \mathcal{E}} V(\Delta \phi_{ij})W_{ij}\right),$$ \hfill(12)

where $\mu$ is a constant ($\mu > 0$), $\mathcal{E}$ is the set of edges connecting two horizontal or vertical neighboring pixels (Bioucas-Dias and Valadão, 2009); $i$ and $j$ represent the two neighboring points; $W_{ij} \in [0, 1]$ is a nonnegative weight, representing the quality of the instantaneous phase data; $V(\Delta \phi_{ij})$ is the potential function, which is a real function defined as follows:

$$V(\Delta \phi_{ij}) = |\Delta \phi_{ij}|^p.$$ \hfill(13)

In equation 13, $\Delta \phi_{ij} = \varphi_i - \varphi_j$, and the exponent $p \in [0, 2]$ is a key parameter for the graph-cut phase unwrapping method. When $0 \leq p < 1$, the method is excellent at preserving discontinuity (Bioucas-Dias and Valadão, 2007; Valadão and Bioucas-Dias, 2009); but when $p \geq 1$, the discontinuity-preserving ability of the method weakens gradually. Therefore, the method is most suitable for the case of $0 \leq p < 1$. In this paper, $p$ is defined between 0 and 1 ($0 \leq p < 1$). Bioucas-Dias and Valadão (2007) and Valadão and Bioucas-Dias (2009) present some specific definitions of the potential function.

MAP estimation

Based on the Bayesian theorem, the posterior probability $p(\Phi | \Psi)$ can be deduced from equations 11 to 13

$$p(\Phi | \Psi) \propto \exp\left(-\mu \sum_{i,j \in \mathcal{E}} V(\Delta \phi_{ij})W_{ij} + \sum_{i \in \mathcal{V}} \lambda_i \cos(\varphi_i - \varphi_i)\right).$$ \hfill(14)

Take a negative logarithm on both sides of above equation, and define its right side as an energy function, notated as $E(\Phi | \Psi)$:

$$E(\Phi | \Psi) = \sum_{i \in \mathcal{V}} -\lambda_i \cos(\varphi_i - \varphi_i) + \mu \sum_{i,j \in \mathcal{E}} |\Delta \phi_{ij}|^p W_{ij}. $$ \hfill(15)

Then, the MAP estimation in equation 5 turns into a problem of minimizing the energy function $E(\Phi | \Psi)$ in equation 15:

$$\Phi = \arg \min_{\Phi} E(\Phi | \Psi).$$ \hfill(16)

Consider the relationship between the absolute and wrapped phase data

$$\Phi = \Psi + 2K\pi, K \in \mathbb{Z},$$ \hfill(17)

where $K$ is an integer data set with the same size and dimension as the wrapped phase data set ($\Psi$). Then phase unwrapping turns into a process to find the integer data set $K$ by minimizing the energy function

$$\tilde{K} = \arg \min_{K} E(\Phi | \Psi),$$ \hfill(18)

where $E(\Phi | \Psi)$ can be expressed as

$$E(\Phi | \Psi) = \sum_{i,j \in \mathcal{E}} |\Delta \phi_{ij}|^p W_{ij} + |\Delta \phi_{ij}|^p W_{ij}.$$ \hfill(19)

Because $-\lambda_i$ is independent of $k_i$ ($k_i \in K$),

$$E(\Phi | \Psi) \equiv \sum_{i,j \in \mathcal{E}} |\Delta \phi_{ij}|^p W_{ij}.$$ \hfill(20)

In a 2D situation, $E(\Phi | \Psi)$ can be decomposed as

$$E(\Phi | \Psi) \equiv \sum_{i,j \in \mathcal{E}} |\Delta \phi_{ij}|^p W_{ij} + |\Delta \phi_{ij}|^p W_{ij},$$ \hfill(21)

where $W_{ij}^h$ and $W_{ij}^v$ are weights representing horizontal and vertical quality information of the phase data, respectively; $\Delta \phi_{ij}^h$ and $\Delta \phi_{ij}^v$ are given by $\Delta \phi_{ij}^h = 2\pi(k_{i+1,j} - k_{ij}) + \varphi_{i+1,j} - \varphi_{ij}$, and $\Delta \phi_{ij}^v = 2\pi(k_{i,j+1} - k_{ij}) + \varphi_{i,j+1} - \varphi_{ij}$. For the sake of notational simplicity, $E_{ij}^h$ and $E_{ij}^v$ are defined by

$$E_{ij}^h(k_{i+1,j}, k_{ij}) = W_{ij}^h|2\pi(k_{i+1,j} - k_{ij}) + \varphi_{i+1,j} - \varphi_{ij}|^p,$$ \hfill(22)

and

$$E_{ij}^v(k_{i,j+1}, k_{ij}) = W_{ij}^v|2\pi(k_{i,j+1} - k_{ij}) + \varphi_{i,j+1} - \varphi_{ij}|^p.$$ \hfill(23)

Therefore, equation 18 can be written as

$$\tilde{K} = \arg \min_{K} E(\Phi | \Psi) \equiv \arg \min_{K} \sum_{i,j \in \mathcal{E}} E_{ij}(k_i, k_j).$$ \hfill(24)

Optimization algorithm

As proven by Bioucas-Dias and Valadão (2007), the minimization of the energy function (equation 24) can be solved by a finite number of binary minimizations:

$$\tilde{K}_{n+1} = \arg \min_{K} E(\Phi | \Psi),$$ \hfill(25)

where $n$ represents the $n$th binary minimization, and $\delta \in \{0, 1\}$ is a binary label list with the same size and dimensions as the phase data.
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set. Each binary minimization (equation 25) can be regarded as a process to solve for the binary label list \( \delta \). Then, the integer data set \( \mathbf{K} \) can be estimated through a finite number of iterative calculations: \( \mathbf{K}^{n+1} = \mathbf{K}^n + \delta \). Therefore, phase unwrapping can be treated as a process to iteratively renew the phase data by adding \( 2\pi \delta \) according to the binary label list \( \delta \):

\[
\Phi^{n+1} = 2\Phi^n + \Psi = \Phi^n + 2\pi \delta.
\] (26)

Substituting \( k_{i,j}^{n+1} (k_{i,j}^n + \delta_{i,j}) \) into equations 22 and 23, and rearranging gives

\[
E_{i,j}(\delta_{i,j+1}, \delta_{i,j}) = W_{i,j}|2\pi(\delta_{i,j+1} - \delta_{i,j}) + d^{|p}|, \tag{27}
\]

and

\[
E_{i,j}^c(\delta_{i+1,j}, \delta_{i,j}) = W_{i,j}|2\pi(\delta_{i+1,j} - \delta_{i,j}) + d^{|p}|, \tag{28}
\]

where \( d^p = 2\pi (k_{i,j+1}^n - k_{i,j}^n) + \phi_{i,j+1} - \phi_{i,j} \) and \( d^c = 2\pi (k_{i+1,j}^n - k_{i,j}^n) + \phi_{i+1,j} - \phi_{i,j} \). Thereby, each binary minimization (equation 25) can be expressed by

\[
\hat{\mathbf{K}}^{n+1} = \arg \min \sum_{i,j \in \mathcal{E}} E_{i,j}(\delta_{i}, \delta_{j}). \tag{29}
\]

According to equations 27 and 28, there are four cases for the \( E_{i,j}(\delta_{i}, \delta_{j}) \): \( E_{i,j}(0,0) = W_{i,j}|d_{i,j}|^p \), \( E_{i,j}(1,1) = W_{i,j}|d_{i,j}|^p \), \( E_{i,j}(0,1) = W_{i,j}|2\pi + d_{i,j}|^p \), and \( E_{i,j}(1,0) = W_{i,j}|2\pi + d_{i,j}|^p \).

The binary minimization problem (equation 29) can be solved by using a max-flow/min-cut algorithm with high efficiency (Greig et al., 1989; Veksler, 1999; Boykov et al., 2001; Boykov and Kolmogorov, 2004; Kolmogorov and Zabih, 2004). It consists of the following steps. The first step is to define a graph \( \mathcal{G} = (\mathcal{V}, \mathcal{E}) \), including two assumed terminals (source or sink, noted as \( S \) or \( T \), respectively), a set of the nodes (\( \mathcal{V} \)) corresponding to the phase data points, and a set of edges (\( \mathcal{E} \)) connecting two first-order neighboring nodes or a node and a terminal (Figure 1). Each edge in a graph (\( e_{i,j}, e_{i}^c \) or \( e_{i}^p \)) is assigned with a cost (\( c_{i,j}, c_{i}^c \) or \( c_{i}^p \)) in Figure 1a) is assigned with a cost (\( c_{i,j}, c_{i}^c \) or \( c_{i}^p \)).

According to equations 29, the cost for the edge \( e_{i,j}, c_{i,j} \), is defined as \( E(0,1) - E(1,0) \); if \( E(1,1) - E(0,0) > 0 \), the cost for the edge \( e_{i}^c, c_{i}^c \), is defined as \( E(0,1) - E(1,0) \); if \( E(1,1) - E(0,0) < 0 \), the cost for the edge \( e_{i}^p, c_{i}^p \), is defined as \( E(0,1) - E(1,0) \); if \( E(1,0) - E(1,1) < 0 \), the cost for the edge \( e_{i}^c, c_{i}^c \), is defined as \( E(0,1) - E(1,0) \); if \( E(1,1) - E(1,0) > 0 \), the cost for the edge \( e_{i}^p, c_{i}^p \), is defined with the cost \( E(1,0) - E(1,1) \) (Figure 1a).

Once a graph is constructed, a cut with the minimum cost, the so-called "min-cut" will be found. The cost of a cut is defined as the sum of the costs of all the edges which are dissected by the cut. A min-cut is a line in a 2D situation (see the thick dotted line in Figure 1b) or a surface in a 3D situation to separate a graph into two parts: one is located on the source side, and the other is on the sink side. Therefore, a binary label list \( \delta \) can be generated by assigning the nodes separated to the source side with a value of 0, and those separated to the sink side with a value of 1.

After a finite number of binary optimizations, the absolute phase can be estimated by

\[
\hat{\phi}_i = \phi_i + 2\pi k_i, \quad \hat{k}_i \in \mathbb{Z}. \tag{30}
\]

Assume the estimated integer \( \hat{k}_i \) is exact, then \( \hat{\phi}_i = \phi_i \) according to equation 7. Therefore, denoising is necessary for a phase unwrapping method due to the existence of noise in real phase data \( \phi_i \). In general, a filtering operation is performed to the wrapped phase data \( \Psi \). An obvious defect of the operation is that those true discontinuities, caused by the abrupt variations of the objects (e.g., unconformities or faults in seismic data), are easily damaged by smoothing before the phase unwrapping. In the graph-cut method (Valadão and Bioucas-Dias, 2009) adopted in this paper, however, denoising is executed after the phase unwrapping. It is achieved by using a multiprecision optimization strategy, which covers a spectrum of optimization precisions from coarse to fine. The optimizations with coarser precisions are operated first to fulfill phase unwrapping, while the succeeding optimizations with finer precisions are carried out for denoising. The major advantage of the multiprecision optimization strategy is that the optimizations with coarser precisions have good discontinuity-preserving capability and thus the true discontinuities are effectively preserved in the absolute phase data (Valadão and Bioucas-Dias, 2009).

Improvements of the method

In this paper, seismic horizon and unconformity constraints are introduced into the process of seismic phase unwrapping, which greatly enhanced the applicability of the graph-cut phase unwrapping method. The horizon constraints can improve the reliability of an unwrapped result by ensuring the correct result on the constraining horizons and avoiding the possible error propagation crossing the horizons. The unconformity constraints can guarantee that the phase unwrapping method runs in a geologically reasonable way at an unconformity. In addition, the original 2D method presented by Valadão and Bioucas-Dias (2009) (called “the original method”)

Figure 1. Construction of a graph (modified from Valadão and Bioucas-Dias, 2009). (a) An elementary graph including two terminals (a source and a sink, which are noted as \( S \) or \( T \), respectively), two adjacent nodes (\( V_i \) and \( V_j \)) and three edges \( (E_{i,j}, e_{i}^c \) and \( e_{i}^p \)). (b) A graph consisting of several elementary graphs, including two terminals (\( S \) and \( T \)), a set of the nodes corresponding to the phase data points, and a set of edges connecting two first-order neighboring nodes or a node and a terminal. The thick dotted line is an assumed min-cut separating the graph into two parts.
Horizon constraints

The first term on the right side of the objective function in equation 15 is called the data fidelity term; it is used to measure the mismatch between the observed data \( q_i \) and the estimated absolute phase \( \phi_i \) (Valadão and Bioucas-Dias, 2009). It penalizes the deviation of the estimated phase from the observed phase, but it respects the consistency between the estimated and observed phases. The data fidelity term varies between \(-1\) and \(1\), according to equation 15. When \( \phi_i \) satisfies the condition in equation 1, the estimated phase is suggested to fit the observed phase, and the data fidelity term is assigned with a minimum penalty value, equal to \(-1\).

Without constraints from the data fidelity term, the estimated result will be unreasonable. However, when heavy noise or large observation errors exist, overemphasizing the consistency between the estimated result and the observed data would skew the estimations.

In this paper, horizon constraints, instead of using the data fidelity term, are introduced into the phase unwrapping to ensure the estimated phase fits the observed phase. The idea of horizon constraints is based on the fact that the phase values are equal everywhere along a continuous and conformable seismic event or horizon, which is suggested to represent a geologically isochronal surface (Vail et al., 1977). Therefore, the unwrapped phase along a horizon should be equal everywhere, which suggests that all the points on the horizon should be assigned with the same labels (0 or 1) in each binary minimization; that is, the horizon should not be dissected by a min-cut. Horizon constraints are exactly introduced to prevent a tracked horizon from being dissected by a possible inaccurate min-cut. In our method, only the reliable events or segments of events are picked out as guidance to the phase unwrapping. Here, “reliable” events refer to those with strong amplitude and high continuity, and without reflection terminations (onlap, downlap, toplap, and truncation) which are easy to track automatically. By using our autotracking method, the events in regions with good data quality will be picked out, and those with poor data quality will not be tracked.

The horizon constraints are incorporated into the process of the construction of a graph. They are achieved by assigning an edge that links two adjacent nodes on the same horizon, with a cost of infinity to indirectly prevent a horizon from being cut by a min-cut; the latter always tends to cut those edges with small costs.

For two adjacent nodes on a horizon, there are two possible ways of linking them to form an edge. If they are vertically or horizontally adjacent, as shown in Figure 2a and 2b, respectively, they can be directly connected to form an edge (\( e_{i,j}^v \) or \( e_{i,j}^h \)) and the cost (\( c_{i,j}^v \) or \( c_{i,j}^h \)) for the edge is designated as infinity. However, if the two nodes are diagonally adjacent (Figure 2c to 2f), they cannot be directly connected to form an edge under the assumption of the first-order Gauss-Markov random field. In this case, two edges, one vertical and one horizontal, are needed to define the connection between them as shown in Figure 2c to 2f. The vertical and horizontal edges (\( e_{i,j}^v \) and \( e_{i,j}^h \)) linked to the diagonal edge are assigned with the same cost of infinity. For two adjacent nodes \( v_i \) and \( v_j \) that are not on the same horizon, the cost of the edge linking them can be defined according to Figure 1.

The 2D seismic profile shown in Figure 3 is an example to demonstrate the effectiveness of horizon constraints. Figure 3a is the original seismic instantaneous phase or wrapped phase profile. Most of the events on the profile are continuous and easy to be tracked; but in some regions, reflections are chaotic and vague, and few continuous horizons can be identified. In addition, two discernible unconformities occur on the upper part of the profile. Those chaotic regions and unconformities pose challenges for a phase unwrapping method. Figure 3b shows the horizons and unconformities used as constraints. Figure 3c and 3d displays the unwrapped results by the original method without horizon constraints and our improved method with horizon constraints, respectively. Although both unwrapped results (Figure 3c and 3d) reflect the general stratal configurations, some disturbance and disruption of events and many singular values are observed in the result by the original method without horizon constraints, which is unreasonable in geology. Figures 3e and 3f will be discussed later in the “Reliability test of the RGT results” section.

To compare the results more closely, two small rectangles in Figure 3 are enlarged in Figures 4 and 5, respectively. Figure 4 displays the results of the upper red rectangle in Figure 3. From the enlarged wrapped phase profile shown in Figure 4a, some chaotic and discontinuous reflections are observed in the middle segment of

**Figure 2.** Six cases for horizon constraints. (a) Two nodes on the same horizon are vertically adjacent; (b) two nodes on the same horizon are horizontally adjacent; and (c-f) two nodes on the same horizon are diagonally adjacent. Considering the first-order neighboring nodes, two diagonally adjacent nodes cannot be directly connected to form an edge. In this case, two edges, one vertical and one horizontal, are needed to define the connection between them. The dotted lines represent the horizons, and the black lines with arrows indicate the edges connecting two first-order neighboring nodes. The costs for all the edges are assigned with infinity.
the profile. Figure 4b shows the unwrapped phase profile by the original method without horizon constraints. In the figure, the originally continuous events both above and below the middle chaotic segment are distorted, and some unreasonable anomalous patches, jumps, and false lateral discontinuities are observed, which are ascribed to the influences of the middle chaotic segment. In contrast, the result by our improved method with horizon constraints, as shown in Figure 4c, displays the overall features in accordance with the wrapped phase image (Figure 4a), and the originally continuous events are not affected by the poor-quality data at the middle segment, indicating that our method with horizon constraints has good noise immunity. It is also noted that the continuity of the events at the discontinuous segment are apparently improved, which is a by-product of horizon constraints.

Figure 5 shows the enlarged images of the lower black rectangle in Figure 3. In the rectangle, the reflections in the upper part are discontinuous or chaotic, while those in the lower part are continuous and layered (Figure 5a). Figure 5b is the unwrapped results by using the original method without horizon constraints. In the middle of the figure, a vertical fault is noted, which is actually not present in the corresponding original wrapped phase images (Figure 5a). This false fault is suggested to be caused by the errors propagating from the upper poor-quality data area to the lower good-quality data area. On the contrary, the result by our method with horizon constraints (Figure 5c) displays good correspondence to the original wrapped phase image (Figure 5a), and no false fault is observed (Figure 5c).

In summary, horizon constraints can ensure the phase data corresponding to the input constraining horizons are correctly unwrapped, and thus can effectively avoid the propagation of errors from the poor-quality data areas with discontinuous or chaotic reflections to the good-quality data areas with continuous or layered reflections. Therefore, by using horizon constraints, the reliability of unwrapped phase results can be greatly improved. Without constraints from horizons, a reasonable and reliable phase unwrapping result is difficult to generate, especially when discontinuous or chaotic reflections develop.

Unconformity constraints

Consider a continuous sedimentary succession without any unconformity. Most phase unwrapping methods can usually work well to generate a reasonable RGT volume from the corresponding seismic instantaneous phase data. Unfortunately, stratal unconformities are ubiquitous. According
to a principle of geology, an unconformity represents a discontinuity in geologic time. Crossing an unconformity, there is a break or hiatus in geologic time. Recovering the discontinuities in geologic time is a challenge to a phase unwrapping method, which calls for a strong discontinuity-preserving ability for a phase unwrapping method. What’s worse, incorrect phase unwrapping at unconformities would result in numerous discontinuities of the unwrapped result in the conformable data areas. Therefore, how to properly deal with the unconformities is a key problem which must confront every phase unwrapping method.

The graph-cut phase unwrapping method adopted in this paper is known for its good discontinuity-preserving ability (when \( 0 \leq p < 1 \)) (Valadão and Bioucas-Dias, 2009). Nevertheless, it is difficult to accurately recover the discontinuities in geologic time at an unconformity without any constraints from the unconformity. In this paper, the interpreted unconformities or sequence boundaries are introduced as constraints, which significantly improve the performance of the phase unwrapping method.

It is a good strategy to improve the discontinuity-preserving ability of a phase unwrapping method by using a reliable phase quality map (if available). Generally, a quality map can be extracted from a correlation map or a phase derivative variance map of the wrapped phase data. Low correlation or high derivative variance suggests poor data quality. The quality control is implemented by weighting the potential function of the objective function (\( W_{i,j} \) in equation (15)). In a general process of quality control, the weight \( W_{i,j} \) is a real number between 0 and 1. Our unconformity constraints can be regarded as a special kind of quality control. It is a binary weighting method in which the weight \( W_{i,j} \) is taken as 0 or 1; \( W_{i,j} = 0 \) implies a discontinuity in the corresponding absolute phase (Bioucas-Dias and Valadão, 2007), indicating the existence of an unconformity; \( W_{i,j} = 1 \) means that no discontinuity exists. Using the unconformity constraints, the wrapped phase samples at an unconformity are excluded from the optimization, and a phase profile (or volume in the 3D case) will be separated into a series of islands by the curves (or surfaces in the 3D case) with zero weights corresponding to the constraining unconformities. Therefore, the discontinuities are restricted at the unconformities, and the unwrapped phase between two adjacent islands will contain arbitrary gaps of \( 2\pi \) (Flynn, 1997), which represent discontinuous jumps in relative geologic time between the upper and lower strata separated by an unconformity. The discontinuity in unwrapped phase data corresponding to the unconformity would be automatically constructed after the phase data above and below the unconformity are correctly unwrapped.

To ensure that the downward relative geologic time jumps crossing an unconformity are nonnegative, that is, the phase values below an unconformity should be larger than those above the unconformity (assuming all the strata above an unconformity are younger than those below), a strategy is presented by which the phase data are updated according to a revised label list. It is different from the original method presented by Valadão and Bioucas-Dias (2009), in which the phase data are updated directly according to the label list generated in each binary optimization. Figure 6 shows the process of revising our label list. Figure 6a is an assumed binary label list generated by a binary minimization according to equation 25. Figure 6b is the label list revised from the one shown in Figure 6a by adding one to all the labels below the unconformity, if there are one-valued labels right above an unconformity. After the label list is revised, the phase data will be updated according to it. By doing so, we can ensure that the unwrapped phase value below an unconformity is always bigger than those above it.

The seismic profile in Figure 7a was used to test the effectiveness of our unconformity constraints. In the profile, two obvious angular unconformities exist (see the white dotted lines in Figure 7a). Figure 7b shows the unwrapped result using the original method without horizon and unconformity constraints. No relative geologic time jump is found at the two unconformities, suggesting the original method fails to recover the geologic time discontinuities at the unconformities accurately. What’s worse, a lot of lateral jumps of relative geologic time, indicating discontinuities, appear in the conformable depositional area (see the white dotted circles in Figure 7b), which is obviously geologically unreasonable. Figure 7c is the result of our method with horizon and unconformity constraints in which the jumps of relative geologic time at the unconformities are clearly discerned. The diachronic property along the unconformities, indicated by the increase of relative geologic time along the unconformities from left to right, is also noted.

Figure 5. Enlarged images of the lower black rectangle on the profile in Figure 3. (a) The enlarged seismic instantaneous phase (wrapped phase) profile. (b) The enlarged unwrapped phase image using the original method presented by Valadão and Bioucas-Dias (2009) without horizon constraints \((p = 0.6)\). (c) The enlarged unwrapped phase image by our improved method with horizon constraints \((p = 0.6)\).

Figure 6. (a) An assumed binary label list generated by a binary minimization in phase unwrapping. (b) The revised binary label list from the one shown in (a) by adding all the labels below the unconformity (marked by a thick line) with one if there are one-valued labels right above an unconformity.
Extending the method from 2D to 3D

The original graph-cut phase unwrapping method presented by Valadão and Bioucas-Dias (2009) is 2D. However, the data sets confronted in modern seismic interpretation are mostly 3D. Therefore, it is necessary to extend the method from 2D to 3D to meet the needs of modern 3D seismic interpretation.

In a 2D problem, only the horizontal and vertical neighbors are considered when the objective function (equation 21) was established under the assumption of a first-order Markov random field. In a 3D situation, however, three-directional (x, y, and z) first-order neighbors should be considered in the construction of the objective function. Then, the objective function becomes

\[
E(K|Ψ) \propto \sum_{i \in V} |Δφ_i|^p W_i^p + |Δφ_i'|^p W_i'^p
\]

where the \( Δφ_i \), \( Δφ_i' \), and \( Δφ_i'' \) are given by

\[
Δφ_i = 2π(k_{i+1}^x - k_i^x) + φ_i^{t+1} - φ_i^t, \quad Δφ_i' = 2π(k_{i+1}^y - k_i^y) + φ_i^{t+1} - φ_i^t, \quad Δφ_i'' = 2π(k_{i+1}^z - k_i^z) + φ_i^{t+1} - φ_i^t.
\]

Here, Ψ is a 3D wrapped seismic instantaneous phase data volume, K is a 3D integer data set with the same size and dimensions as the instantaneous phase data volume.

In the 3D case, all the issues should be considered in a 3D perspective, including the establishment of a graph in each binary minimization, the definition of the edges linking two first-order neighbors, and the definition of the costs for the edges, as well as the introduction of horizon and unconformity constraints. Finally, the 3D phase unwrapping method will generate a superior solution to its 2D counterpart because phase information from 3D space is utilized in the process of 3D phase unwrapping.

For a huge 3D seismic data cube, the memory requirement and computation time will be a challenge to our 3D phase unwrapping method and all the methods based on the \( L^p \) norm minimization. There are two possible strategies to improve computation efficiency. One way is to reduce the size of the data cube by increasing the spatial sampling interval. For example, only use every third CDP and every third line. The other way is to subdivide the data cube into smaller manageable blocks. Costantini (1998) uses blocks with partial overlapping of data and then performs the phase unwrapping sequentially for each block. Stark (2005c) discusses using either adjoining or overlapping blocks that are unwrapped independently (sequentially or parallel) and then the results are reconciled to form a single consistent and contiguous RGT volume.

**APPLICATION**

The 3D improved graph-cut phase unwrapping method with horizon and unconformity constraints presented in this paper is applied to a 3D seismic survey in northern Sichuan Basin, southwest China to test the robustness of the method. In the area, late Permian reef and early Triassic ooid carbonate reservoirs are the most important exploration targets. By using traditional seismic sequence analysis procedures (Vail et al., 1977), seven seismic sequence boundaries are identified, which, from bottom to top, are named SB1 to SB7 (Figure 8a), respectively. They classified the late Permian and early Triassic stratal succession into six seismic sequences (Figure 8a). These sequence boundaries (SB1–SB7) are used as unconformity constraints.

Figure 7. (a) The wrapped phase profile in which two obvious unconformities are marked with white dotted lines. (b) The unwrapped result using the original method presented by Valadão and Bioucas-Dias (2009) without unconformity constraints (\( p = 0.6 \)). (c) The unwrapped result by our improved phase unwrapping method with unconformity constraints (\( p = 0.6 \)). (d) The first vertical difference profile of the RGT result generated by the original method. (e) The first vertical difference profile of the RGT result generated by our improved method, in which two color-changing lines can be observed; their colors changing from left to right represent the two unconformities with hiatuses increasing from left to right.
constraints. In addition, the reliable strong seismic events picked by a 3D autotracked method are used as horizon constraints.

The test survey is 400 lines × 500 traces (10 × 12.5 km) in area. In the test, only the phase data covering the target interval between SB1 and SB7 is used, which is contained into a window of 800 ms in length (400 samples for each trace). Thus, the input data size is equal to 400 × 500 × 400 in samples. For our phase unwrapping algorithm, the total memory required is about 10 G, and the computation time is four hours for a 2.53 GHz PC.

Figure 8 displays the results of a 2D profile extracted from the 3D data volume, including the interpreted sequence boundaries as unconformity constraints (Figure 8a); the wrapped seismic phase section with superimposed black lines indicate the autotracked horizons used as horizon constraints (Figure 8b), and the corresponding unwrapped phase section (Figure 8c). Figure 9 is the 3D visualization of the test survey, including the original seismic amplitude data volume (Figure 9a) and the unwrapped RGT volume (Figure 9b). From the unwrapped results (Figures 8c and 9b), it can be observed that the relative geologic time within a single sequence gradually increases downward as the traveltime increases, and the relative geologic time at the sequence boundaries displays discontinuous jumps. The reflection terminations indicating the sequence boundaries, including onlap, downlap, toplap and erosional truncation, are also clearly revealed.

On the basis of an RGT volume, a geologically isochronous slice corresponding to a desired relative geologic time can be extracted by connecting the samples with the same relative geologic time (Stark 2003, 2004a, 2004b, 2005a). These slices are similar to the stratal slices produced by Zeng et al. (1998a, 1998b), but when using an RGT volume our slices can contain hiatus locations which are unavailable from Zeng et al.’s method. As an example, a set of isochronous slices were extracted from the RGT volume in the test survey. Figure 10 shows the positions of the slices on a vertical north–south-extending profile (line section). In the figure, the red thick lines represent the interpreted sequence boundaries used as unconformity constraints, and the thin lines with different colors correspond to the extracted isochronous slices. To demonstrate the slices in the map view, six representative slices, one for each sequence, are selected, which are marked in thick blue lines in Figure 10. The superimposed isochronous slices in Figure 10b match well with the seismic events on the vertical profile, suggesting our RGT result is reasonable.

Figure 11 displays the absolute amplitude maps of the selected slices. In the figure, the absolute reflection amplitude for each slice extends in a northwest–southeast direction, which is in accordance with the regional depositional trend. Regional sedimentological research suggests that the upper Permian to lower Triassic strata in the test survey formed in a northwest–southeast-extending carbonate platform environment, and they consist of, from southwest to northeast; open platform, platform margin, frontal slope, and basin facies belts, respectively. Some geomorphological details are noted in the maps, which require further investigations and are beyond the scope of this paper.

**DISCUSSION**

**Geologic time significance of wrapped and unwrapped phases**

Generation of an RGT volume by phase unwrapping is built on the concept that instantaneous or unwrapped phase is related to relative geologic time (Stark, 2003, 2004a, 2005c). According to the principle of seismic stratigraphy (Vail et al., 1977), continuous seismic reflections occur primarily on the bedding surfaces and unconformities with sufficient velocity-density contrasts. The former are the relics of paleodepositional surfaces, and thus represent geologically synchronous surfaces; the latter have chronostratigraphic significance, in that all rocks below an unconformity are older than those above it (Vail et al., 1977). Therefore, primary seismic reflections or horizons have chronostratigraphic significance.

The exploration seismology principle tells us that the instantaneous or wrapped phase is identical along a conformable seismic event, which suggests that the instantaneous or wrapped phase carries information of geologic time (Stark, 2003). Therefore, the unwrapped phase obtained by accurate phase unwrapping processing should be identical along a conformable seismic event. This forms the basis of our horizon constraints.

In addition, consider a vertical conformable stratigraphic sequence whose geologic time increases downward; its unwrapped phase will gradually increase downward. If an unconformity occurs in the
succession, there will be a hiatus in geologic time crossing the unconformity and the unwrapped phase corresponding to the unconformity will display a jump. In our method, the unconformities interpreted are introduced as guidance to ensure the phase unwrapping is reliable at the unconformities.

In real complicated seismic data, some phase changes are not related to geologic time, such as those related with flat spots, bottom simulating reflectors, faulting fractured zone, and so on. In these cases, our horizon constraints are proven to be especially useful, by preventing the possible RGT errors from propagating across the constraining horizons.

Figure 9. Visualization of the test survey. (a) The original seismic amplitude data volume in which the target interval between SB1 and SB7 is used as the input data. (b) The 3D unwrapped result generated by our improved 3D graph-cut phase unwrapping method with horizon and unconformity constraints ($p = 0.7$).

Figure 10. The same north–south-extending profile as shown in Figure 8, displaying (a) the positions of a set of isochronous slices, extracted from the generated RGT volume, on the profile, and (b) the seismic amplitude profile superimposed with the sequence boundaries (red lines) and the selected six isochronous slices (blue lines). The thick red lines indicate the interpreted sequence boundaries, the thin lines with different colors represent the isochronous slices extracted, and the thick blue lines represent the selected isochronous slices, as shown in Figure 11.

Figure 11. Six absolute amplitude isochronous slices extracted from the generated RGT volume in the test survey, one slice for each sequence. (a–f) Slices 1 to 6, extracted from sequences 1 to 6, respectively. See Figure 10b for locations of the slices on the vertical profile.
Phase unwrapping in regions with poor data quality

In real seismic data volumes, low-quality data inevitably exist, which may result from heavy noise or complex geologic factors including faults, facies changes, and so on. How to deal with them is a challenge for a successful use of the phase unwrapping method. We use two methods to overcome this challenge.

First, the seismic data are preprocessed using a structure-constrained filtering algorithm (Fomel, 2002; Fehmers and Höcker, 2003; Hale, 2009, 2011) before the horizons’ autopicking and phase unwrapping are performed. By this preprocessing step, most of the noise can be removed.

Another method is to use a mask map to guide the phase unwrapping. It turns out to be an effective approach for a phase unwrapping method to tackle noisy data by masking out the low-quality phase data or those violating the Itoh condition (Goldstein et al., 1998). A mask map is a special quality map in which a data point takes a value of one or zero. In a mask map, zero-valued points mark low-quality phase data that will be zero-weighted and masked out, and one-valued points mark high-quality ones that will be weighted with one during phase unwrapping (Goldstein et al., 1998). A mask map can be constructed using a correlation map or a phase derivative variance map (Goldstein et al., 1998). In this paper, we adopt a derivative variance map of seismic instantaneous phase data to construct a mask map. When the derivative variance value of a point is bigger than a given threshold value, it is designated with zero in the mask map, otherwise it is designated as one. The poor-data quality points or those violating the Itoh condition (equation 2) are marked by zero-valued zones in the mask map because they are usually of large derivative variances. Figure 12 is an example to demonstrate the effect of the mask map for low-quality data. Figure 12a is the original seismic instantaneous phase profile. Figure 12b is the mask map of the profile (Figure 12a) in which the poor-data quality regions including the fault and facies change zones are successfully masked out.

In addition, the graph-cut phase unwrapping method adopted in the paper is known for its good noise-immunizing performance. As far as we know, it is the only phase unwrapping method with self-denoising performance (Valadão and Bioucas-Dias, 2009). That is one of the major reasons why we choose the graph-cut method to fulfill phase unwrapping of seismic data.

Processing of faults

How to deal with faults is a big challenge for phase unwrapping. In this paper, we use a way to tackle faults similar to what we do for unconformities because faults also represent a kind of discontinuity. Like in the unconformity constraints described above, we deal with a fault by weighting the potential function of the data points on the fault with zero ($W_{ij} = 0$ in equation 15).

Figure 12 is an example to demonstrate the performance of our method in a seismic profile developed with faults, as shown in Figure 12a. In the example, four obvious normal faults are observed (Figure 12a). The faults are detected automatically by using the mask map method described above (Figure 12b). Cohen et al. (2006) present a more robust method to automatically detect the faults. In the RGT profile shown in Figure 12c, the four normal faults are successfully revealed by using the faults as guidance of phase unwrapping. It should be pointed out that, for some complicated faults that are difficult to detect automatically, manually interpreted faults need to be introduced to ensure a reasonable phase unwrapping result.

Reliability test of the RGT results

To test the reliability of our phase unwrapping results, two methods are used: one is the seismic data flattening based on an RGT result, and the other is a first vertical difference profile of an RGT result.

![Figure 12. The phase unwrapping in a 2D seismic profile complicated by faults. (a) The seismic instantaneous phase profile in which the black and green dotted lines indicate the reference events dislocated by the faults. (b) The mask map of the phase profile generated from a derivative variance map of the phase profile shown in (a). (c) The RGT result generated by our method with the mask map control. (d) The flattened result of the seismic phase profile based on the RGT result shown in (c).](image-url)
Seismic data flattening based on an RGT result is an approach to generate a 2D chronostatigraphic chart or a 3D chronostatigraphic volume (Stark, 2005a). It is also a simple and direct method to test the reliability of the RGT result generated by phase unwrapping. If an RGT result is unreasonable, conformable seismic events cannot be flattened according to their corresponding RGT values. In this case, the seismic events will not be a horizontal line (or surface) on the resulted chronostatigraphic chart (or volume). As an example, Figure 12d displays the flattened seismic instantaneous phase profile based on its corresponding RGT result (Figure 12e) generated by our method. In the figure, most of the seismic events are reasonably flattened (see the seismic events marked by black and green dotted lines). The segments belonging to the events dislocated by the faults are also correctly reassembled on the flattened profile (Figure 12d). In addition, the few chaotic seismic events at the bottom of the profile, which may be the result of spatial facies changes, are also more or less flattened. This example suggests, to a certain degree, the reliability of our method.

As suggested by T. J. Stark (personal communication, 2011), the first vertical difference profile of RGT values provides a good test of the reliability of the RGT result using phase unwrapping. In principle, the RGT value will increase downward, and thus, all the values in the vertical difference profile should be positive if the phase unwrapping result is reasonable. Negative values on a vertical difference profile indicate the existence of errors in RGT results. Figures 3e, 3f, 7d, and 7e are examples of the first vertical difference profiles of RGT values, in which Figures 3e and 7d are obtained by the original method, and Figures 3f and 7e are generated by our method. In the figures, blue points represent the negative values indicating RGT errors. It is noted that the blue points generated in Figures 3f and 7e by our method are much less than those in Figures 3e and 7d by the original method, which suggests the robustness of our method. In addition, these RGT errors can be further reduced by using a mask map to guide the phase unwrapping.

CONCLUSIONS

On the basis of comparative analysis of current methods, the graph-cut phase unwrapping method is introduced into the unwrapping of seismic instantaneous phase data to generate an RGT volume. Based on the Bayesian theorem, the method treats the phase unwrapping as a problem of maximum a posteriori (MAP) estimation, which is achieved by a series of binary minimizations. Each binary minimization includes the construction of a graph and the succeeding finding of a min-cut to separate all the nodes in the graph into two parts. A binary label list \( \delta \) is then generated by assigning the nodes in one part with a value of 0, and those in the other part with 1. Therefore, in this method, the phase unwrapping is achieved through a series of updating from the wrapped phase by adding a series of \( 2\delta \). The method has the advantages of wide adaptability (suitable for \( L^p \) norm problems with \( 0 \leq p \)), strong discontinuity-preserving ability (when \( 0 \leq p < 1 \)), and high efficiency of computation.

Constraints from seismic horizons are successfully introduced into the phase unwrapping process of seismic data. The idea of horizon constraints is based on the fact that continuous seismic events or horizons are of chronostratigraphic significance, which suggests that the unwrapped phase on a horizon should be constant. Horizon constraints are introduced by setting the costs of the edges linking two adjacent nodes on the same horizon with infinite values, which can indirectly prevent a min-cut from separating a horizon in each binary optimization. Therefore, the nodes on the same horizon are assigned with the same label in each binary optimization, and then those nodes on the same horizon will share the same updating step in each binary optimization, which eventually results in a constant unwrapped phase on the same horizon. In practice, the strong seismic events, which can be easily tracked by certain autopicking methods, are used as the constraining horizons.

Discontinuities of the absolute phase caused by ubiquitous unconformities are difficult to recover by a phase unwrapping method. In this paper, the interpreted unconformities or sequence boundaries are introduced as constraints to improve the discontinuity-preserving ability of the phase unwrapping method to correctly recover the discontinuities of relative geologic time at an unconformity. The unconformity constraints are achieved by a binary weighting strategy, in which the wrapped phase data on the constraining unconformities are weighted with 0 and the others with 1. By this strategy, the wrapped phase data on the constraining unconformities are excluded in each binary optimization, which restricts the discontinuities of the wrapped phase data at the unconformities.

In the paper, the original 2D graph-cut phase unwrapping method of Valadão and Bioucas-Dias is extended into a 3D one to meet the needs of 3D seismic interpretation. By using the 3D phase unwrapping method, we can obtain a more reliable solution superior to its 2D counterpart.

As a test, our improved 3D graph-cut phase unwrapping method is applied to a real 3D seismic survey in northern Sichuan Basin, southwest China. An RGT volume is obtained, from which a set of isochronous slices are generated. The isochronous slices match well with the seismic events. The absolute amplitude maps of the slices reveal a northwest–southeast-extending trend of facies belts, which is in accordance with the regional depositional trend. All those suggest the robustness of our improved graph-cut phase unwrapping method.

The construction of the potential function in our phase unwrapping method is based on the approximate assumption of a first-order Gauss-Markov random field to the phase data set, and only the first-order neighboring points are considered. A better phase unwrapping result is expected if higher-order neighboring points are considered in the construction of the potential function. In addition, the estimated orientation information of seismic events can be used to constrain the construction of the potential function, which may improve the noise immunity of our phase unwrapping method.

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