Viscoelastic time-reversal imaging

Tieyuan Zhu

ABSTRACT

The time invariance of wave equations, an essential precondition for time-reversal (TR) imaging, is no longer valid when introducing attenuation. I evaluated a viscoelastic (VE) TR imaging algorithm based on a novel VE wave equation. By reversing the sign of the P- and S-wave loss operators, the VE wave equation became time invariant for the TR operation. Attenuation effects were thus compensated for during TR wave propagation. I developed the formulations of VE forward modeling and TR imaging. I tested my imaging approach in three numerical experiments. The first experiment used a 2D homogeneous model with full-aperture receivers to examine the time invariance of the VE TR imaging equation. Using the same model, the second experiment was used to demonstrate the method’s ability to characterize a point source. In the third experiment, I applied this method to characterize a complex source using borehole geophones. Numerical results illustrated that the VE TR imaging improved our knowledge of the source location, radiation pattern, and amplitude.

INTRODUCTION

Using the time-invariance and reciprocity properties of wave equations, time-reversal (TR) algorithms are able to retrace the recorded wave propagation path back through the medium and converge on the location of initial sources. The robustness and simplicity of TR techniques make them effective tools for resolving source localization problems in numerous physical fields including seismology (e.g., Kremers et al., 2011), land-mine detection (e.g., Norville et al., 2004), nondestructive detection (e.g., Saenger, 2011), and acoustics (e.g., Fink, 2006).

When intrinsic attenuation is considered in the earth, however, wave equations are no longer time invariant under TR (Fink, 2006). As a result, TR waves that are backpropagated into such a medium under the same conditions as in a nonattenuating medium lose the property of symmetry in time. The complication of time variance can be addressed by an appropriate compensation for the intrinsic attenuation applied to the backward-propagated wavefields during TR propagation (Gosselet and Singh, 2007; Zhu, 2014). By compensating in this way for attenuation effects during TR propagation, we should expect to reconstruct the backward-propagated wavefield in amplitude and phase and properly resolve the final focused source.

My previous work proposed to compensate for attenuation effects in the viscoacoustic TR process (Zhu, 2014). Using a novel viscoacoustic wave equation that decouples attenuation and dispersion effects, viscoacoustic TR modeling becomes time invariant upon reversing the sign of the P-wave loss operator. In other words, attenuation effects were compensated and the approach produced a general improvement in source magnitude and resolution. Furthermore, the viscoacoustic TR modeling is implemented in attenuation-compensated reverse-time migration (Zhu et al., 2014) and TR source imaging (Zhu, 2014).

Elastic TR imaging can locate a seismic source by backpropagating seismograms from receivers to find where the energy focuses. Elastic TR imaging is able to reconstruct not only the source location but also the radiation pattern (possibly the source mechanism) (Hu and McMechan, 1988; Steiner et al., 2008; Kremers et al., 2011). Because the source mechanism is usually determined from the phase and amplitude of direct waves, neglecting attenuation may lead to incorrectly inverted source mechanisms for elastic waves (Eisner et al., 2011).

In this paper, I introduce a viscoelastic (VE) TR imaging approach that can correct for both P- and S-attenuation effects. Like the viscoacoustic wave equation, this VE wave equation also has the property of decoupled attenuation-associated loss and dispersion operators for P- and S-waves (Zhu and Carcione, 2014). In terms of this property, a VE TR modeling equation is formulated by reversing the sign of the P- and S-attenuation operators and therefore is time invariant. I demonstrate these features of the VE TR approach with two synthetic models.
METHODOLOGY

In this section, I discuss TR imaging in the framework of VE wave propagation in inhomogeneous media. First, I briefly introduce the VE wave equation and then the propagation equation for TR imaging.

Viscoelastic forward modeling

The frequency-independent $Q$ model (i.e., constant-$Q$ model) is considered to be a practical approximate $Q$ model for exploration seismology problems (e.g., Zhu et al., 2014). Based on the mathematical constant-$Q$ model (Kjartansson, 1979), the 2D velocity-stress formulation of the VE wave equation is given by Zhu and Carcione (2014):

$$\rho \partial_t u_1 = (\partial_1 \sigma_{11} + \partial_3 \sigma_{13} + f_1),$$

(1)

$$\rho \partial_t u_3 = (\partial_1 \sigma_{13} + \partial_3 \sigma_{13} + f_3),$$

(2)

$$\sigma_{11} = [\eta_p B_p (\epsilon_{11} + \epsilon_{33}) - 2\eta_s B_s \epsilon_{33}]$$

$$+ \left[ \tau_p A_p \frac{\partial}{\partial t} (\epsilon_{11} + \epsilon_{33}) - 2\tau_s A_s \frac{\partial}{\partial t} \epsilon_{33} \right],$$

(3)

$$\sigma_{33} = [\eta_p B_p (\epsilon_{11} + \epsilon_{33}) - 2\eta_s B_s \epsilon_{11}]$$

$$+ \left[ \tau_p A_p \frac{\partial}{\partial t} (\epsilon_{11} + \epsilon_{33}) - 2\tau_s A_s \frac{\partial}{\partial t} \epsilon_{11} \right],$$

(4)

$$\sigma_{13} = 2\eta_s B_s \epsilon_{13} + 2\tau_s A_s \frac{\partial}{\partial t} \epsilon_{13},$$

(5)

and

$$\epsilon_{11} = \partial_1 u_1, \quad \epsilon_{33} = \partial_3 u_3, \quad \epsilon_{13} = \frac{1}{2} (\partial_3 u_1 + \partial_1 u_3),$$

(6)

where $\rho$ is the mass density; $\nu$, $\sigma_i$, $\epsilon_{ij}$, and $f_i$ denote the particle velocity tensor, the stress tensor, the strain tensor, and body force components, respectively; $u_i$ is the displacement; $v_i = \partial_i u_i$; and $i, j$ are spatial indices. Einstein’s convention of repeated indices is assumed. And,

$$A_{P,S} = (-\nabla^2)^{\nu_{P,S}}/2, \quad B_{P,S} = (-\nabla^2)^{\gamma_{P,S}},$$

(7)

where

$$\tau_p = C_p c_0^{2\nu_p - 1} \sin(\pi \gamma_p), \quad \eta_p = C_p c_0^{2\nu_p} \cos(\pi \gamma_p),$$

$$\tau_s = C_s c_0^{2\nu_s - 1} \sin(\pi \gamma_s), \quad \eta_s = C_s c_0^{2\nu_s} \cos(\pi \gamma_s),$$

$$C_{\lambda} = M_0 \omega_0^{-2\gamma_p}, \quad C_{\mu} = \mu_0 \omega_0^{-2\gamma_s},$$

(8)

and $\omega_0$ is an arbitrary reference frequency, which should be higher than the source frequencies to guarantee pulse delay with respect to the lossless case. Also, $\gamma_{P,S} = a \tan(Q_{P,S}^3)/\pi$ and $0 < \gamma_{P,S} < 0.5$ for any positive value of $Q$, where $Q_p$ and $Q_p$ are the P- and S-wave quality factors, respectively. The P-wave modulus $M_0$ and the S-wave modulus $\mu_0$ are, respectively, given by $M_0 = pc_0^2 \cos^2 (a \tan(Q_p^3)/2)$ and $\mu_0 = pc_0^2 \cos^2 (a \tan(Q_s^3)/2)$, and $c_0$ and $c_0$ are the P- and S-wave velocities at the reference frequency, respectively.

To solve the wave equation in inhomogeneous media, I use the staggered-grid finite-difference approach to discretize the time derivatives and the staggered-grid pseudospectral approach to discretize the first-order spatial derivatives. The fractional Laplacian operators are solved by the fractional Fourier pseudospectral method as shown by Zhu and Harris (2014).

Note that the first-order time-derivative terms in equations 3–5 correspond to attenuation-associated P- and S-wave loss operators. When $Q_{P,S} \to \infty$ ($\gamma_{P,S} \to 0$), the first-order time-derivative terms disappear, and equations 1–5 only contain second-order time derivatives and become an elastic wave equation that is time-invariant under TR. When $Q_{P,S}$ is finite, this VE propagation equation contains the first-order time-derivative loss operators and, therefore, invariance under TR is lost (Fink, 2006).

Viscoelastic time-reversal modeling/imaging

To implement TR in the time-domain wave equation mathematically, I first replace time $t$ by $-t$. Due to attenuation during forward propagation, I have to amplify amplitude during the propagation of the TR wavefields. The amplification of amplitude is done by reversing the sign of the P-wave loss operator (Zhu, 2014). For the VE case, I reverse the sign of P- and S-wave loss operators. Assuming $i = -t$, the VE TR modeling equations with attenuation compensation are written as

$$\sigma_{11} = [\eta_p B_p (\epsilon_{11} + \epsilon_{33}) - 2\eta_s B_s \epsilon_{33}]$$

$$- \left[ \tau_p A_p \frac{\partial}{\partial t} (\epsilon_{11} + \epsilon_{33}) - 2\tau_s A_s \frac{\partial}{\partial t} \epsilon_{33} \right],$$

(9)

$$\sigma_{33} = [\eta_p B_p (\epsilon_{11} + \epsilon_{33}) - 2\eta_s B_s \epsilon_{11}]$$

$$- \left[ \tau_p A_p \frac{\partial}{\partial t} (\epsilon_{11} + \epsilon_{33}) - 2\tau_s A_s \frac{\partial}{\partial t} \epsilon_{11} \right],$$

(10)

and

$$\sigma_{13} = 2\eta_s B_s \epsilon_{13} + 2\tau_s A_s \frac{\partial}{\partial t} \epsilon_{13},$$

(11)

Substituting equations 9–11 into equations 1 and 2 (with $i = -t$), we can see that the solution $u(x, -t)$ of equations 1, 2, 9, 10, and 11 is the TR version of the solution $u(x, t)$ of the forward modeling equations 1–5, where $x = (x, y, z)$. The waveform at a forward time $t$ is physically equivalent to that at the reversed time $-t$. Therefore, the above system for TR modeling becomes time invariant.

For the reverse modeling, the input data consist of two particle velocity components (vertical and horizontal). No body force is present throughout the TR propagation; i.e., $f_i = 0$. The recorded particle velocity components are reversed in time and enforced as the Dirichlet boundary condition at receivers; mathematically they are expressed as
\[ v_1(x_r, t) = u_z(T - t), \quad \text{and} \quad v_3(x_r, t) = u_z(T - t). \] (12)

Here, \( v_1 \) and \( v_3 \) are the recorded horizontal and vertical particle velocity components at the receivers and \( T \) is the total recorded time. The receiver location coordinate in 2D is \( x_r = (x_r, z_r) \).

In practice, the higher frequencies in the recorded data are invariably contaminated with noise. Attenuation compensation during TR propagation will amplify such unwanted frequency content. To prevent high-frequency noise from growing exponentially, I apply a low-pass filter in the spatial frequency domain to the right-hand-side (RHS) attenuation-associated loss and dispersion operators in equations 9–11 when calculating the TR wavefields. The filter should not be applied to the propagation operators. Equations 9–11 become

\[
\begin{align*}
\sigma_{11} &= \Omega + F[\eta_\nu B_P(\epsilon_{11} + \epsilon_{33}) - 2\eta_\sigma B_S(\epsilon_{33} - \Omega)]
- F \left[ \tau_P A_p \frac{\partial}{\partial t}(\epsilon_{11} + \epsilon_{33}) - 2\tau_S A_S \frac{\partial}{\partial t} \epsilon_{33} \right], \quad (13) \\
\sigma_{33} &= \Psi + F[\eta_\nu B_P(\epsilon_{11} + \epsilon_{33}) - 2\eta_\sigma B_S(\epsilon_{33} - \psi)]
- F \left[ \tau_P A_p \frac{\partial}{\partial t}(\epsilon_{11} + \epsilon_{33}) - 2\tau_S A_S \frac{\partial}{\partial t} \epsilon_{11} \right], \quad (14) \\
\sigma_{13} &= \Theta + F[2\eta_\sigma B_S(\epsilon_{13} - \Theta)] - F \left[ 2\tau_S A_S \frac{\partial}{\partial t} \epsilon_{13} \right], \quad (15)
\end{align*}
\]

where \( F \) indicates the designed filter in the wavenumber domain because these equations are discretized by the pseudospectral method. The first bracket in the RHS represents the attenuation-associated dispersion, and the second bracket represents the loss. The propagation operators are

\[
\begin{align*}
\Omega &= [C_\lambda(\epsilon_{11} + \epsilon_{33}) - 2C_\mu \epsilon_{33}], \\
\Psi &= [C_\lambda(\epsilon_{11} + \epsilon_{33}) - 2C_\mu \epsilon_{11}], \quad \text{and} \quad \Theta = 2C_\mu \epsilon_{13}. \quad (16)
\end{align*}
\]

Note that, if there is no attenuation, the last two terms in the RHS of equations 13–15 disappear. Thus, equations 13–15 become elastic. A Tukey-window-shaped filter was chosen in the numerical examples. The cutoff wavenumber is calculated by the cutoff frequency divided by the maximum velocity of the medium (Zhu, 2014).

The particle velocity and stress components are determined and stored at every time step and every computational grid within this VE forward and TR modeling algorithm. I choose the maximum value of sum of the stress tensor as an imaging condition (Saenger, 2011). This imaging condition is written as

\[
\text{image} = \max_{i,j} \sum \sigma_{ij}(x, t). \quad (17)
\]

**SYNTHETIC EXAMPLE**

In my first example, I use my VE TR modeling approach to characterize a point source in a 2D homogeneous model. The P-wave velocity is 2500 m/s, the S-wave velocity is 1500 m/s, the density is 2200 kg/m³, \( Q_P = 40 \), and \( Q_S = 20 \). The model is discretized with 256 × 256 grid points. The grid spacing of the horizontal and vertical axes is \( \Delta x = \Delta z = 6 \text{ m} \). The source is at the origin point. Its center frequency is 25 Hz with a time delay of 0.04 s. The time step is \( 4.8 \times 10^{-4} \text{ s} \). The horizontal and vertical particle velocities are recorded through time with 400 receivers, which are located in a circle with radius 600 m centered at the source (Figure 1).

I generated synthetic VE data with a horizontal single force. Then I ran the elastic and VE TR imaging approaches. The Tukey filter was chosen with a cutoff frequency of 120 Hz and a taper ratio of 0.4. As a reference image, I also ran elastic TR imaging with simulated elastic data, i.e., without any attenuation effects.

**Wavefield reconstruction by VE TR**

In the first test, I show the time invariance of the VE TR approach; i.e., the waveform at a specified point in the computational grid can be reconstructed during VE TR propagation. Here, I have two reference points A and B for all simulations in Figure 1.

Figure 2 shows reconstructed waveforms at reference points A and B. Due to attenuation between the reference points and receivers, the waveform generated by elastic TR imaging (dashed black line) is spread out (phase shifted) and has a reduced amplitude at A and B compared with the reference waveform (black line). We can see that the more attenuated S-waves have larger shifts than the P-waves. Those incorrect waveforms at A and B reinforce that the attenuation breaks the time invariance of TR propagation. The reconstructed waveform by VE TR imaging with attenuation compensation (gray line) is comparable with the reference waveform in amplitude and phase for P- and S-waves. The recovery of the waveform demonstrates the time invariance of the TR propagation equation for the VE TR imaging. The small difference between the black and gray lines might be numerical error caused by the finite-difference discretization.

![Figure 1](image_url)
Characterizing the source

My second test characterizes the point source using TR imaging. Figure 3a-3c show TR images in the pressure magnitude snapshot. Three TR images give the source location and, roughly, the source area as well as the source radiation pattern. However, Figure 3b gives an incorrect radiation pattern that is opposite in sign to the reference one. Using VE TR imaging, Figure 3c presents not only the correct radiation pattern but also an improved source amplitude estimate. These observations can also be easily identified from the horizontal (Figure 3d) and vertical (Figure 3e) cross sections through the origin. The dominant wavelength is about 100 m (P-wave velocity of 2500 m/s and central frequency of 25 Hz). The source area is sharper in Figure 3c with an adequate accuracy of 100 m or smaller (see the blue lines in Figure 3d and 3e).

In the third test, I apply VE TR imaging to characterize a complex source. Here, I consider a downhole geometry for microseismic monitoring. The P-wave velocity and $Q_p$ models are shown in Figure 4. I assume $V_S = V_P/1.7$ and $Q_S = Q_P/2$. The model is discretized with $814 \times 641$ grid points. The grid spacing of the horizontal and vertical axes is $\Delta x = \Delta z = 0.2$ m. The source-receiver geometry is plotted in Figure 4. The injection well is assumed on the left side, and the lowest velocity around depth 100 m models the effects of fluid injection. Microseismic sources are represented by modeling 57 individual sources (vertical force) with center frequency ranging between 600 and 1000 Hz (i.e., a Ricker wavelet). These sources are randomly distributed within the white-dashed-line area in Figure 4. The monitoring well is on the right. The horizontal and vertical particle velocities are recorded through time with 25 geophones deployed at depths ranging from 39 to 136 m with a spacing of 4 m. A time step of 11 μs was used. Elastic and VE synthetic data were generated by the VE forward modeling approach described in the above section. The Tukey filter was chosen with a cutoff frequency of 2000 Hz and a taper ratio of 0.4.

Figure 5 presents the TR imaging results of the VE data without (b and e) and with (c and f) attenuation compensation. For comparison, I also reconstructed the source location and magnitude using elastic TR modeling of elastic data, with the results appearing in Figure 5a and 5d). It is noteworthy that without attenuation compensation, the magnitude of the source is underestimated and the imaged source area is shifted about one dominant wavelength (about 5 m) from the designed source area (dashed line zone). When using VE TR modeling, the estimated source location is corrected. The source radiation pattern in Figure 5f agrees with the reference in Figure 5d very well. The sharpness and magnitude of the reconstructed source is also notably improved. These observations can also be easily identified from the cross sections at 100-m depth in Figure 6. The spatial resolution has been clearly enhanced.

**DISCUSSION**

The results I have presented indicate that the source location, radiation pattern, and amplitude can be corrected using the VE TR imaging approach. The improvements in the source loca-

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**Figure 2.** Waveforms (horizontal particle velocity component) recorded at the reference points A (top) and B (bottom). Black waveform recorded in the forward simulation using VE forward modeling; dashed line, waveform reconstructed by elastic TR modeling; and gray, waveform reconstructed by VETR modeling in this paper. The recovery of the waveform (gray) demonstrates that the equation for the VE TR imaging approach is time invariant. The labels “P” and “S” denote the P- and S-waves, respectively.

**Figure 3.** Focused image of the horizontal force source by (a) elastic TR imaging with elastic seismograms; (b) elastic TR imaging with VE seismograms; and (c) VETR imaging with VE seismograms. Observe that elastic imaging does not even recover the correct sign of the amplitude. (d) Horizontal and (e) vertical cross sections of focused images through the origin are compared.

**Figure 4.** I assume $V_S = V_P/1.7$ and $Q_S = Q_P/2$. The model is discretized with $814 \times 641$ grid points. The grid spacing of the horizontal and vertical axes is $\Delta x = \Delta z = 0.2$ m. The source-receiver geometry is plotted in Figure 4. The injection well is assumed on the left side, and the lowest velocity around depth 100 m models the effects of fluid injection. Microseismic sources are represented by modeling 57 individual sources (vertical force) with center frequency ranging between 600 and 1000 Hz (i.e., a Ricker wavelet). These sources are randomly distributed within the white-dashed-line area in Figure 4. The monitoring well is on the right. The horizontal and vertical particle velocities are recorded through time with 25 geophones deployed at depths ranging from 39 to 136 m with a spacing of 4 m. A time step of 11 μs was used. Elastic and VE synthetic data were generated by the VE forward modeling approach described in the above section. The Tukey filter was chosen with a cutoff frequency of 2000 Hz and a taper ratio of 0.4.

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tion, radiation pattern, and amplitude provide hope that we may be able to invert the seismic moment tensor by VE TR source imaging (Kawakatsu and Montagner, 2008). There are other potential applications in seismology and exploration seismology. For example, in microseismic experiments, attenuation seriously affects the accuracy of the source location and the amplitudes of P- and S-waves, especially in surface recordings (Eisner et al., 2011). Attenuation corrections could increase the frequency bandwidth and improve the signal-to-noise ratio. VE TR imaging using correct P- and S-wave attenuation promises to improve the accuracy of the microseismic source location, radiation pattern, and amplitude, which provide the quantitative basis of source mechanisms. Detailed investigation of the quantitative relation between the source mechanisms and the source image by VE TR imaging is needed.

It is also worth noting that the VE TR modeling approach can also reconstruct the propagated wavefield at any arbitrary point in attenuating media. A direct application is the use of the VE TR approach for prestack VE reverse time migration to reconstruct the receiver wavefield. The reconstructed receiver wavefield would be more similar in amplitude and location compared with the source wavefield. Crosscorrelating the compensated source and receiver wavefields could improve the image resolution of reflectors (Zhang et al., 2010; Zhu et al., 2014). It is also potential to implement this method in the VE full-waveform inversion with recomputing the forward wavefield in reversed time from the receiver positions instead of the one based on checkpoints in which the forward wavefield is recomputed forward in time from these checkpoints to possibly save the computational costs (Tarantola, 1988; Blanch and Symes, 1995; Symes, 2007; Anderson et al., 2012).

Finally, numerous studies have shown that TR imaging is theoretically connected to seismic interferometry (e.g., Derode et al., 2003; Bakulin and Calvert, 2006). When attenuation is significant,
due to time variance, seismic interferometry with crosscorrelation methods yields a Green’s function with incorrect traveltimes and amplitudes (Snieder, 2007). The principle of reconstructing the wavefield by VE TR imaging could shed light on seismic interferometry in attenuating media.

CONCLUSIONS

I have presented a VE TR imaging approach. The TR propagation equation will satisfy time invariance by reversing the sign of P- and S-wave loss operators while at the same time compensating for the P- and S-wave attenuation effects in the reconstructed wavefield and source. Numerical results show that my TR imaging approach is able to reconstruct the wavefield with the correct phase and amplitude at an arbitrary point in the media and also improves the accuracy of the source location, radiation pattern, and amplitude. This method might be particularly advantageous for characterizing microseismic sources, reverse time migration, and full-waveform inversion.

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