Time-reverse modelling of acoustic wave propagation in attenuating media

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SUMMARY

Time-reverse modelling (TRM) of acoustic wave propagation has been widely implemented in seismic migration and time-reversal source imaging. The basic assumption of this modelling is that the wave equation is time-invariant in non-attenuating media. In the Earth, attenuation often invalidates this assumption of time-invariance. To overcome this problem, I propose a TRM approach that compensates for attenuation and dispersion effects during the wave propagation in attenuating media. This approach is based on a viscoacoustic wave equation which explicitly separates attenuation and dispersion following a constant-$Q$ model. Compensating for attenuation and dispersion during TRM is achieved by reversing the sign of the attenuation operator coefficient while leaving the counterpart dispersion parameter unchanged in this viscoacoustic wave equation. A low-pass filter is included to avoid amplifying high-frequency noise during TRM. I demonstrate the effects of the filter on the attenuation and the phase velocity by comparing with theoretical solutions in a 1-D Pierre shale homogeneous medium. Three synthetic examples are used to demonstrate the feasibility of attenuation compensation during TRM. The first example uses a 1-D homogeneous model to demonstrate the accuracy of the numerical implementation of the methodology. The second example shows the applicability of source location using a 2-D layering model. The last example uses a 2-D cross-well synthetic experiment to show that the methodology can also be implemented in conjunction with reverse-time migration to image subsurface reflectors. When attenuation compensation is included, I find improved estimation of the source location, the excitation timing of the point source, the magnitude of the focused source wavelet and the reflectivity image of reflectors, particularly for deep structures underneath strongly attenuating zones.

Key words: Seismic attenuation; Computational seismology; Wave propagation.

INTRODUCTION

In general, time-reverse modelling (TRM) of acoustic wave propagation consists of two steps: (1) reversing recorded seismic data in time and (2) backpropagating the time-reversed data as sources from receiver locations through an appropriate earth model. This method has been successfully implemented in many applications in seismology and exploration geophysics. For example, time-reversal source imaging was suggested by McMechan (1982) to backpropagate seismograms from receivers to find the focusing energy, that is, the location of sources. Further applications have since been reported for the determination of temporal and spatial parameters of earthquake sources (McMechan et al. 1985; Kao & Shan 2004; Lu 2007; Kremers et al. 2011), earthquake source patterns (Hu & McMechan 1987), moment tensor inversion (Kawakatsu & Montagner 2008; O’Brien et al. 2011), tremors (Larmat et al. 2006; Lokmer et al. 2009) and location of an active seismic source (Gajewski & Tessmer 2005). More recently, TRM has been applied to locate microseismic sources for reservoir monitoring (e.g. Lu et al. 2008; Steiner et al. 2008; Larmat et al. 2009). Another example is reverse-time migration (RTM) which adds two additional steps to the TRM procedure: propagating the wavefield from the source and further applying an imaging condition to obtain the structure interface (Baysal et al. 1983; McMechan 1983). These studies, whether using simple acoustic or even elastic anisotropic wave equations, did not consider attenuation in their forward and backward (reverse) modelling.

In a non-attenuating medium, a wave equation is time-invariant for TRM. In other words, with perfect receiver sampling, the backward-propagated wavefield mirrors the forward-propagated wavefield in time. In the 1-D experiment shown in Fig. 1(a), we can see that the final focused energy is able to reconstruct the initial source, which is physically supported by the assumption of a lossless medium (Fink 2006).

However, when intrinsic attenuation is considered in the Earth, the wave equation is no longer time-invariant under time reversal (Fink...
The purpose of this paper is to develop a rigorous and general TRM approach for attenuating media and to test this TRM approach using synthetic seismic experiments. In our previous study (Zhu & Harris 2014), we derive a time domain, nearly constant-$Q$ viscoacoustic wave equation for modelling wave propagation in attenuating media. This viscoacoustic wave equation can model approximate constant-$Q$ attenuation and dispersion behaviour of seismic waves in the relevant seismic frequency band. More importantly, it decouples attenuation and dispersion effects, a definite advantage for attenuation compensation during TRM (Treeby et al. 2010). Based on this equation, I present an adjoint viscoacoustic wave equation to compensate for attenuation and dispersion effects by simply changing the sign of attenuation operator. The implementation flow for TRM with attenuation compensation is presented in the following section. Next, I show that this TRM procedure will be stabilized by applying a low-pass filter to attenuation and dispersion operators in the spatial frequency domain. Finally, I validate the TRM approach in a 1-D homogeneous model, and show two applications to illustrate the effectiveness of compensating for attenuation and dispersion effects for time-reversal imaging and RTM.

**METHODOLOGY OF TRM IN ATTENUATING MEDIA**

**Constant-$Q$ model**

For seismic modelling and imaging, attenuation (proportional to $1/Q$) is considered to be approximately linear with frequency in many observational frequency bands (McDonal et al. 1958; Kjartansson 1979; Aki & Richards 1980), that is, $Q$ is constant over these frequency ranges. Kjartansson (1979) explicitly gave a linear description of attenuation that exhibits the exact constant-$Q$ characteristic. The dispersive phase velocity and attenuation of the constant-$Q$ model are given by Kjartansson (1979) as follow:

$$c_p = c_0 \left( \frac{\omega}{\omega_0} \right)^\gamma,$$

$$\alpha = \tan \frac{\pi \gamma}{2} \frac{\omega}{c_p},$$

where the velocity $c_0$ is given at a reference frequency $\omega_0$, the parameter $\gamma = 1/\pi \tan^{-1}(1/Q)$ is dimensionless, and we know $0 < \gamma < 0.5$ for any positive value of $Q$. Hence, the quality factor $Q$
is frequency-independent, that is, constant $Q$. Note that constant $Q$ is not exactly equivalent to assuming that $\alpha$ is proportional to frequency since $c_P$ is slightly dependent on frequency.

Forward modelling

Based on the constant-$Q$ model (Kjartansson 1979), the time domain nearly constant-$Q$ (visco)acoustic wave equation in attenuating media was first introduced by Zhu & Harris (2014). It is written as

$$\frac{1}{c_0^2} \frac{\partial^2 p_B}{\partial t^2} = \eta L p_B + \tau H \frac{\partial}{\partial t} p_B,$$

(3)

where the source wavelets emit at source positions $(x_s, z_s)$,

$$p_B(x_s, z_s, t) = p_0(x_s, z_s, t),$$

(4)

and the two fractional Laplacian operators are $L = (-\nabla^2)^{1/2}$, $H = (-\nabla^2)^{3/2}$; $p_B$ is the spatial pressure field of forward modelling in time $t$ from 0 to $T$, and $c_0$ is the acoustic velocity at the reference frequency $\omega_0$. The attenuation and dispersion operator coefficients are given by

$$\eta = -c_0^2 \eta_0 \gamma \cos \gamma,$$

and

$$\tau = -c_0^2 \eta_0^{-1} \omega_0 \eta_0^2 \sin \gamma,$$

(5)

where the variable $\gamma$ is defined as $\gamma = 1/\pi \tan^{-1}(1/Q)$. The range of $\gamma$ is $0 < \gamma < 1/2$ with any positive $Q$. The first term in the right-hand side of eq. (3) is related to dispersion effects, and the second is related to attenuation effects (Zhu & Harris 2014). The most attractive feature of this viscoacoustic wave equation is the explicit separation of attenuation and dispersion (eq. 3). This is in contrast to other wave equations in which the attenuation and dispersion are encapsulated by a single term (Deng & McMechan 2007, 2008; Carcione 2010). In particular, in the following section, I show the viscoacoustic wave equation to be advantageous for compensating for attenuation and dispersion effects during TRM.

Time-reverse modelling

Mathematically, time reverse involves replacing time $t$ by $-t$. Therefore, I write the constant-$Q$ wave equation (eq. 3) in reversed time as follows:

$$\frac{1}{c_0^2} \frac{\partial^2 p_B}{\partial (t - \tau)^2} = \eta L p_B - \tau H \frac{\partial}{\partial t - \tau} p_B,$$

(6)

where the boundary condition is

$$p_B(x_s, z_s, t) = p_B(x_s, z_s, T - t).$$

(7)

Here, $p_B$ is the spatial pressure wavefields of TRM in time $t$, and $p_B(x_s, z_s, T - t)$ is the data recorded at time $T - t$ at the receiver positions.

This TRM wave equation is not identical to the forward modelling wave equation (eq. 3), because the first-order time derivative, representing attenuation effects breaks the temporal symmetry of equation (eq. 3). However, if I simply change the sign of the attenuation term in eq. (6), I have

$$\frac{1}{c_0^2} \frac{\partial^2 p_B}{\partial (t - \tau)^2} = \eta L p_B - \tau H \frac{\partial}{\partial t} p_B,$$

(8)

which becomes a time-invariant wave equation for TRM since it is the same as the forward modelling in eq. (3). Thus, with $p_B(x, t)$ as the solution of the forward modelling equation (eq. 3), $p_B(x, -t)$ is a solution of eq. (3) with the sign of the attenuation term reversed.

The negative sign functions to compensate for attenuation during the propagation of the reversed wavefields. I emphasize that the first, dispersion-related term on the right-hand side of eq. (8) is time-independent and does not reverse sign (i.e. the frequency-dependent phase velocity remains unchanged in time). As explicated by Treeby et al. (2010), higher frequencies travel to the receiver faster than lower frequencies in the forward propagation; when waves back-propagate in reversed time, higher frequencies will again need to travel faster than the lower frequencies to arrive simultaneously at the original source. As a result, using this wave equation implicitly compensates for dispersion in wavefields. We will see this in the first numerical experiment.

In practice, the higher frequencies in recorded data are invariably contaminated with noise. Attenuation compensation during TRM might amplify such unwanted frequency content. To prevent high-frequency noise from growing exponentially, I apply a low-pass filter to the attenuation and dispersion operators in eq. (8) in the spatial frequency domain when calculating the time-reversed wavefields. The cut-off wavenumber is calculated by the cut-off frequency over the maximum velocity of media.

To filter out the high-frequency noise caused by attenuation compensation, I rewrite eq. (8) by substituting $\tau = -t$

$$\frac{1}{c_0^2} \frac{\partial^2 p_B}{\partial (t + \tau)^2} = \eta L p_B - \tau H \frac{\partial}{\partial t} p_B,$$

(9)

where $\nabla^2 p_B$ denotes non-attenuated dispersion, $\eta L p_B - \nabla^2 p_B$ denotes attenuation associated dispersion and $\eta L p_B$ is the attenuation operator. By this means, I avoid filtering the first Laplacian operator $\nabla^2 p_B$, which is independent on attenuation. Indeed, eq. (9) is an adjoint constant-$Q$ viscoacoustic wave equation for TRM. If we substitute $-\tau = \tau$ into eq. (9), it becomes eq. (3).

Below, I show the effect of the filter on attenuation and phase dispersion. A Tukey window shaped filter was illustrated for high-frequency noise control. The attenuation and dispersion values are calculated using signals recorded at 20 and 100 m from the source (Treeby & Cox 2010; Zhu & Harris 2014). The medium parameters are given in the first example in the following section. I applied two different Tukey filters in eq. (9). The cut-off frequency was set to 250 Hz, and the taper ratios were 0.5 and 0.1, respectively. Figs 2(a) and (b) show that the calculated attenuation and dispersion curves agree with theoretical curves of Kjartansson’s constant-$Q$ model within the filter passband. Frequencies above our cut-off curves are damped as desired.

Numerical implementation

To incorporate a perfectly matched layer (PML) absorbing boundary condition in the numerical simulation, I chose to use a coupled first-order constitutive equation instead of a second-order equation (eqs 3 and 9). Eq. (9) is thus rewritten as

$$\frac{\partial v}{\partial t} = -\frac{1}{\rho_0} \nabla p,$$

(10)

$$\frac{\partial p}{\partial t} = -\rho_0 \nabla \cdot v,$$

(11)

$$p = c_0^2 \left[1 + (\eta L - 1) - \tau \nabla \cdot v\right] p,$$

(12)

where

$$L = (-\nabla^2)^{1/2}, \quad \nabla \cdot v = (-\nabla^2)^{-1/2},$$

(13)
In the first example, I applied my time-reversal imaging approach to characterize a point source in a 2-D model. The conventional TRM process.

TRM workflow

My complete procedure for implementing TRM in an attenuating medium consists of three steps:

1. Reverse recorded seismic data in time $p_B(x, y, z, T - t)$ and then enforce that data as a boundary condition at those original receiver positions, mathematically expressed by $p_B(x, y, z, \tilde{t}) = p_B(x, y, z, T - \tilde{t})$, where $x, y, z$ are the coordinates of receivers.

2. Solve eqs (10)–(12) with the boundary condition for propagating waves in reversed time. Attenuation and dispersion effects are automatically compensated as time progresses. The source–receiver geometry is usually known in the data. The medium parameters (velocity, density and attenuation) are assumed to be known prior to the TRM process.

3. Finally, search for the maximum value of the pressure field $p_B(x, y, z, \tilde{t})$ at each time step and store the spatial position. The location of the maximum pressure amplitude throughout entire time of TRM is the source position $(x_0, y_0, z_0)$ and its time $t_0$ is the excitation time of the source (Gajewski & Tessmer 2005; Saenger 2011), where $\tilde{t}$ runs from 0 to $T$ and $t_0 = T - \tilde{t}$. The pressure field $p_B(x, y, z, T - t_0)$ is shown as the time-reverse imaging.

APPLICATIONS

Two applications, including 2-D time-reversal imaging and RTM in attenuating media, are discussed in this section.

Time-reversal imaging in a 2-D model

In the first example, I applied my time-reversal imaging approach to characterize a point source in a 2-D model. The conventional
Figure 3. Time-reversal snapshots at 0.8 s (top panels) and at the final focused time (bottom panels) in the same medium as forward modelling. The black lines represent the reconstructed wavefields without attenuation compensation (a and c) and with attenuation compensation (b and d). The red lines are reference signals at forward modelling times $t = 0$ s and $t = 0.2$ s, respectively, in a 1-D homogeneous medium. The two vertical dashed lines indicate the receiver locations. Here, the reconstructed source with maximum amplitude appears too early at $\tilde{t} = 0.98$ s in (c). With attenuation compensation, not only is the exact source excitation time recovered but also the magnitude of the reconstructed source (black line) almost perfectly matches the initial source (red line).

The time-reversal imaging technique has previously been used to successfully locate seismic sources (e.g. McMechan 1982; Gajewski & Tessmer 2005; Steiner et al. 2008). Here, I further investigate whether I can improve the characterization of the seismic source (e.g. the spatiotemporal history of the source) with my method. For clear illustration, I use only acoustic data.

Let us consider a cross-well geometry for microseismic monitoring, which has been employed in practice to monitor hydraulic fracturing (Warpinski 2009; Song & Toksöz 2011). The velocity and $Q$ models are shown in Fig. 5. The injection well is on the left-hand side, and the monitoring well is on the right-hand side. The lowest velocity around depth 100 m models the effects of fluid injection. The model is discretized with $841 \times 641$ gridpoints. I use a time step of $11 \mu$s. The grid spacing of the horizontal and vertical axes are $\Delta x = \Delta z = 0.2$ m. A time-varying Ricker wavelet with center frequency of 1000 Hz is injected as a point source at position $(x_S = 11.8$ m, $z_S = 90$ m). 40 receivers are deployed at depths ranging from 12 to 168 m with a spacing of 4 m. The source–receiver geometry is plotted in Fig. 5(a). Synthetic data were generated by a forward modelling viscoacoustic code, with $Q$ based on the standard linear solid model (Zhu et al. 2013). The resulting acoustic and viscoacoustic seismograms are shown in Fig. 6.

The focused energy is used to determine the location and time where the source was excited, thus providing information about the original source. Fig. 7 shows the time-reversal imaging results of the viscoacoustic data without (c) and with (e) attenuation compensation. For comparison, I also reconstructed the source location and magnitude using the acoustic time-reversal imaging of the acoustic data with the results appearing in Figs 7(a) and (b). It is noteworthy that, without attenuation compensation, the magnitude of the source is underestimated. Picking the maximum pressure amplitude yields an estimated source location at $(7.6$ m, $89$ m), which is shifted about one dominant wavelength from the true source location $(11.8$ m, $90$ m). The corresponding excitation time of the estimated source is $-0.56$ ms (the negative sign indicating that the estimated source excitation time is late compared to the original time). When attenuation compensation is included into TRM, the estimated source location is found at $(12.0$ m, $90.2$ m). The sharpness and magnitude of the reconstructed source is also notably improved. These observations can also be easily identified from the cross-sections through estimated $x_S$ and $z_S$ in Figs 8(a) and (b), respectively. The spatial resolution has been clearly enhanced. Meanwhile, I show comparisons of the recorded time-series at the source without and with compensating attenuation in Fig. 8(c). Observe that the temporal history of the source wavelet with attenuation compensation (blue line) approximates the reference one (red line) fairly well. The estimation without attenuation compensation is further from the true location, and therefore it fails to provide source time information.

In order to examine how this modelling approach performs with noisy data, I added Gaussian noise to the two data sets. The Gaussian noise is generated by creating the signal-to-noise ratio (S/N = 1 dB) on a logarithmic decibel scale (the viscoacoustic data are used as the reference signal). Fig. 9 shows noisy acoustic and viscoacoustic data sets. The seismic events cannot be identified in the noisy viscoacoustic seismogram. Fig. 10 shows the time-reversal imaging results for the noisy viscoacoustic data set without (c) and with (e) attenuation compensation. Again, for comparison I obtained the reference image by backpropagating noisy acoustic data using the
Figure 4. Time-reversal experiments conducted at 0.8 s (top panel), 0.985 s (middle panel) and 1.0 s (bottom panel) in the same medium as forward modelling. In addition to attenuation compensation, I reversed the sign of the dispersion operator. The source focusing occurs at $\bar{t} = 0.985$ s with amplitude being somewhat low.

Figure 5. Seismic models—(a) $P$-wave velocity and (b) $Q$. A point source is located at $x_s = 11.8$ m, $z_s = 90$ m, shown as a black star. The inverted triangles represent 40 receivers. The receiver depths range from 12 to 168 m with a spacing of 4 m.

Figure 6. Recorded seismograms at the receiver well, as computed by (a) the acoustic solver and (b) the viscoacoustic solver.

Reverse-time migration

The second example illustrates the feasibility of TRM with attenuation compensation in an RTM algorithm.

Typically, acoustic RTM includes a three-step procedure of (i) forward propagation of a wavefield from a source through an appropriate velocity model, (ii) backpropagation of the measured data in reversed time through the same model and (iii) superposition of both propagations, using an imaging condition. No attenuation is assumed in this implementation. Mittet et al. (1995) presented a complete procedure to compensate for attenuation in both the source and the receiver wavefields in an attenuating medium. Here, rather than compensating for both the source and receiver wavefields simultaneously, I apply attenuation compensation for the receiver wavefields in step (ii) and the attenuation loss for the source wavefield in step (i) using the method described by Mittet et al. (1995).
Figure 7. Time-reverse source imaging using acoustic and viscoacoustic data without noise. The imaging is the maximum amplitude of the pressure field throughout entire time modelling. (a) Reference image and (b) zoom over the source area. (c) Image without attenuation compensation and (d) zoom over the source area. (e) Image with attenuation compensation and (f) zoom over source area. The colour scale indicates the magnitude of the source. The black star represents the true point source location \((x_s = 11.8\, \text{m}, z_s = 90\, \text{m})\). Without attenuation compensation, the estimated source location is shifted. The inverted triangles represent receivers.
wavefields in step (i) (Deng & McMechan 2007). The last procedure to produce a RTM image is to apply a suitable imaging condition to render the reflectors of the medium. I used a source normalized cross-correlation of the forward-propagated source wavefield and backward-propagated receiver wavefield (Claerbout 1971) given by the following formula:

\[
I(x) = \frac{\int_0^T P_f(x, t) P_b(x, T - t) dt}{\int_0^T P_f(x, t) P_f(x, t) dt}.
\]  (14)

As a result, in the imaging position, the image consists of correlated wavefields, which represents the reflectivity of model. The attenuation compensation tends to recover the amplitude loss and approximate reflectivity energy as in the completely non-attenuating case.

The reflectivity of the cross-well test model is shown in Fig. 13(a). The corresponding velocity and Q values of each zone are listed in Table 1. The constant density is 2.2 g cm \(^{-3}\). The most highly attenuating zone is zone 4, with Q = 30. The model is discretized in a 201 × 601 grid with a grid spacing of \(\Delta x = \Delta z = 0.5\) m. Receivers are in the left-hand well. I deployed 101 sources with depths from 5 to 300 m in the right-hand well, located at \(x = 100\) m. The source is a Ricker wavelet with a centre frequency of 400 Hz. I implemented the forward modelling approach (eq. 3) with a PML boundary of 20 gridpoints. Fig. 12 shows the acoustic and viscoacoustc common-shot-gather data, respectively. The shot is located at 150.5 m depth. Apparently, the reflections are attenuated in the viscoacoustic data. Before backpropagation, the direct wave was muted. We used smoothed velocity and Q models for RTM input. For RTM with and without attenuation compensation, the source and receiver wavefields were saved only at every fourth time step to reduce the volume of data storage. The Tukey filter was chosen with a cut-off frequency of 1200 Hz and a taper ratio of 0.5 to suppress the noise growth during attenuation compensation. After applying the normalized cross-correlation imaging condition to the source and receiver wavefields for all 101 shots, I then stacked them and filtered the stacked RTM image to remove low wavenumber noise.

The reference migration image in Fig. 13(b) shows acoustic data migrated with acoustic RTM. Fig. 13(c) displays the RTM output without attenuation compensation. The image lacks illumination at the boundaries of the highly attenuative zone 4 and at the interfaces below. This is because the diffracted or reflected waves travelling through this zone will be attenuated to such a degree that they are hardly identifiable in the recorded data. When attenuation compensation is included, the reflectors in target zones 4 and 5 appear to be better illuminated as shown in Fig. 13(d).

**DISCUSSION AND CONCLUSIONS**

Whereas the traditional TRM approach typically neglects attenuation effects on wavefields, I have presented a new TRM approach in attenuating media to compensate for those effects of wavefields. This TRM approach is based on the adjoint viscoacoustic wave equation. The most attractive feature of this equation is the explicit separation of amplitude attenuation and phase dispersion. Thus,
Figure 10. Time-reversal imaging using the noisy viscoacoustic data in Fig. 9(b). The imaging is the maximum amplitude of the pressure field throughout entire time modelling. (a) Reference image and (b) the magnified source area. (c) Image without attenuation compensation and (d) the magnified source area. (e) Image with attenuation compensation and (f) the magnified source area. The colour scale indicates the magnitude of the source. Black stars represent the point source location. The estimated source location is shifted when attenuation has not been compensated.
Figure 11. Comparisons among the reconstructed sources by the proposed (blue) and conventional time-reversal technique (black) and the reference source (red). (a) Horizontal cross-line at depth $z_s = 90$ m. (b) Vertical cross-line at distance $x_s = 11.8$ m. (c) Time-series recorded at source location ($x_s = 11.8$ m, $z_s = 90$ m).

Table 1. P-wave velocity, density and quality factors of cross-well model in Fig. 13(a).

<table>
<thead>
<tr>
<th>Zone</th>
<th>$C_p$ (km s$^{-1}$)</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.6</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>2.7</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>3.1</td>
<td>150</td>
</tr>
<tr>
<td>4</td>
<td>2.6</td>
<td>30</td>
</tr>
<tr>
<td>5</td>
<td>2.8</td>
<td>50</td>
</tr>
<tr>
<td>6</td>
<td>3.5</td>
<td>300</td>
</tr>
</tbody>
</table>

attenuation compensation can be done by simply reversing the sign of the amplitude attenuation operator and leaving the sign of the phase dispersion operator unchanged. This is superior to using other wave equations for attenuation compensation. For example, reversing the sign of the attenuation operator will not correctly compensate for dispersion because the amplitude attenuation and phase dispersion are encapsulated by a single operator (Deng & McMechan 2007, 2008; Carcione 2010). Another equation proposed by Zhang et al. (2010) introduces the normalization exponential operator for attenuation compensation, which might compensate for amplitude attenuation and phase dispersion but is more or less ad hoc. Compensating for attenuation is further stabilized by applying a low-pass filter, which avoids amplifying the high-frequency signals during propagation. As a result, this approach becomes stable as illustrated in various numerical experiments.

In the time-reversal imaging experiments with attenuation compensation, we found the time-reversed field focused in the vicinity of the original point source location and at the original focal time. The magnitude of the reconstructed source is amplified and approximates the reference source well, even using noisy data. The method is, therefore, especially attractive for weak onsets which are not detectable on single traces of the network, such as microseismic data sets. In practice, however, it may be challenging to apply attenuation compensation when the noise frequency band overlaps the signal frequency band. In this case, denoising processing must be applied before running a TRM with attenuation compensation.

This approach could also be quite valuable for pre-stack RTM to improve the resolution of images, particularly beneath very strong attenuation areas. The reflectors in the conventional RTM might be dimmed due to high-attenuation geological environments but may be clearly imaged in the attenuation compensated RTM.

The primary contribution of this work is to present a novel TRM approach in attenuating media. I have demonstrated the potentials of this TRM approach for the time-reversal imaging of seismic sources and RTM image in attenuating media. Experiments with these synthetic data clearly demonstrate the ability of the TRM method to improve four aspects: (i) the estimation of the source location, (ii) the excitation timing of the point source, (iii) the magnitude of focused source wavelet and (iv) the reflectivity image of structure beneath high-attenuation zones.

Future work should focus on two aspects. First, we can extend the extension to the 3-D viscoacoustic TR modelling and the viscoelastic TR modelling based on the viscoelastic wave equation (Zhu & Carcione 2013). In addition, we need to test this approach with real data for locating (micro)earthquake sources and imaging the subsurface reflectors by RTM.
Figure 13. (a) Reflectivity model with block numbers referenced in Table 1. For illustration, the inverted triangles denote receivers in the left well. The black stars represent sources in the right well; (b) acoustic RTM of acoustic data; (c) acoustic RTM of viscoacoustic data; (d) $Q$-RTM with attenuation compensation of viscoacoustic data.

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